LOGIC BLOOM · JEE PHYSICS

Vectors & Kinematics

Foundations · **Vectors in full** · Kinematics rapid recap

The vector toolkit every mechanics chapter leans on, built from scratch: addition laws, components, and the dot and cross products, each with a diagram. Then a tight, high-yield kinematics recap, and two heavy Important Questions sets worked in the Core, Working, Trap style.

Vectors, full | Kinematics recap | 17 Worked Questions | Main & Advanced | Fact Sheet

Logic Bloom
logicbloom.co.in

The simplest way to learn science.

Contents

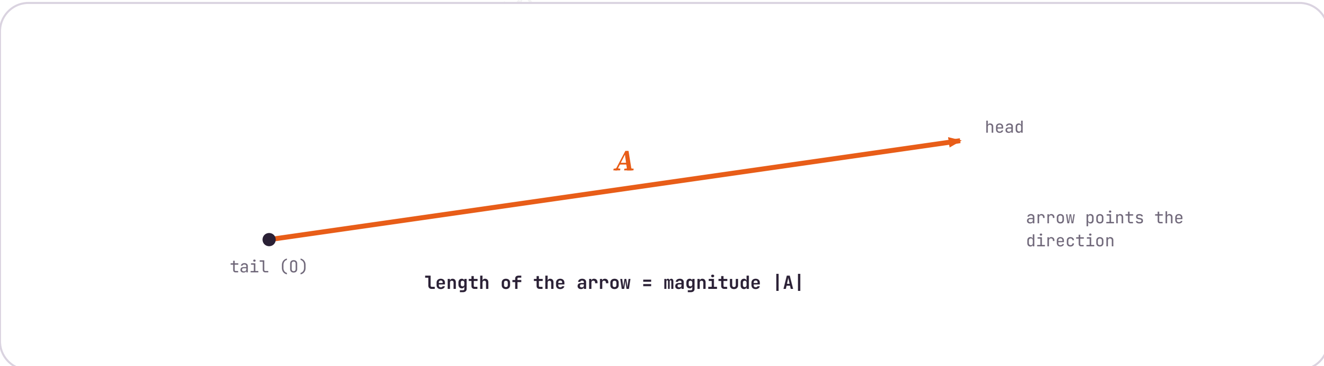
PART A

Vectors

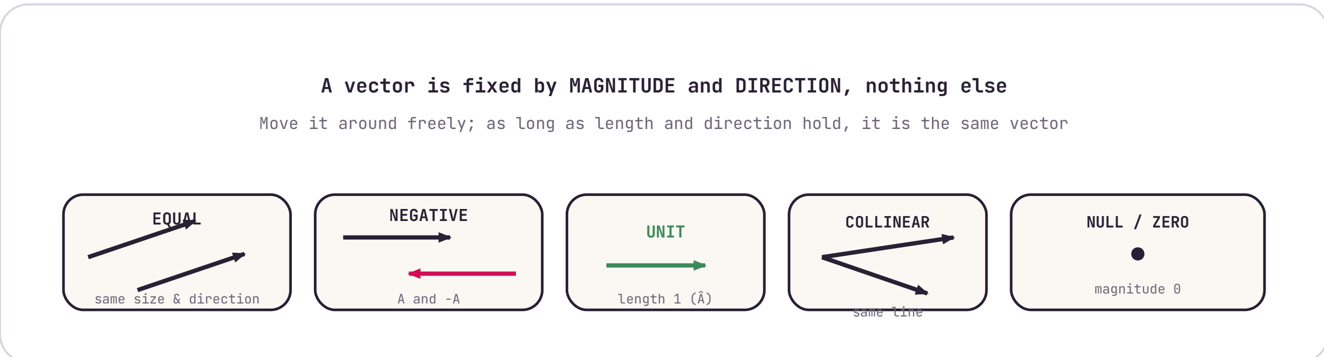
The language of directions, built from the ground up.

1 Scalars and Vectors

A **scalar** has magnitude only (mass, time, speed, energy). A **vector** has magnitude **and** direction (displacement, velocity, force). That extra piece, direction, is why vectors need their own rules of arithmetic.



A vector is drawn as an arrow: its length is the magnitude, the way it points is the direction. Only these two things define it.



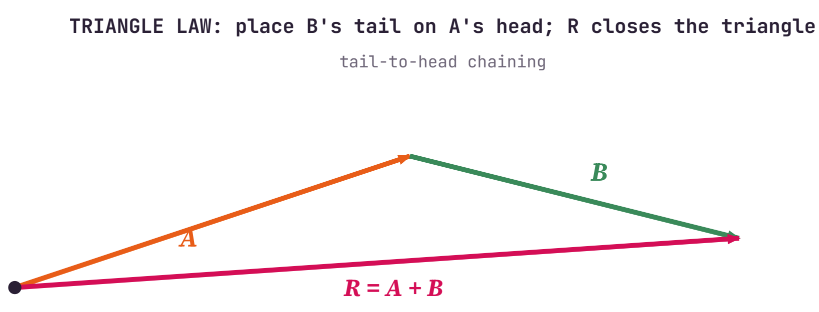
The vocabulary of vectors. A vector can slide anywhere in space; while its length and direction stay fixed, it is the same vector.

TYPES WORTH NAMING

Equal: same magnitude and direction. **Negative:** same size, opposite direction. **Unit:** magnitude 1, written with a hat (Â), used to carry pure direction. **Null:** zero magnitude. **Collinear:** along the same line. **Coplanar:** lying in one plane.

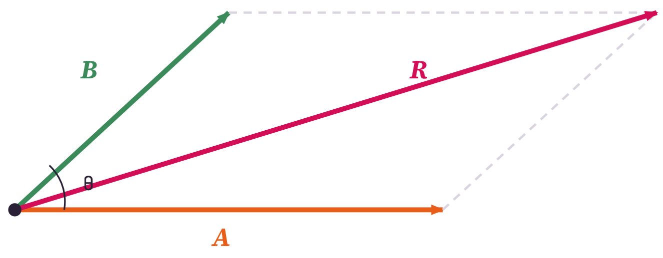
2 Adding Vectors

You cannot add vectors like ordinary numbers; direction matters. Two rules do all the work.



Triangle law: put the tail of the second vector on the head of the first; the resultant runs from the very first tail to the very last head.

PARALLELOGRAM LAW: A and B from the SAME point; R is the diagonal



R = √(A² + B² + 2AB cosθ)

tanα = B sinθ / (A + B cosθ)

Parallelogram law: draw both vectors from a common point; the diagonal is the resultant. The formula gives its exact size and direction.

★ THE RESULTANT OF TWO VECTORS AT ANGLE θ

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

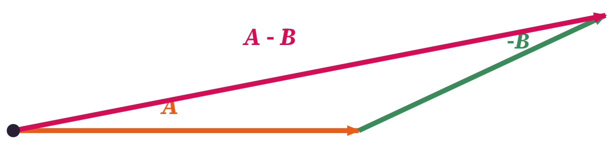
α is the angle the resultant makes with vector A.

TWO SPECIAL CASES

Same direction (θ = 0): R = A + B, the maximum. **Opposite** (θ = 180°): R = |A − B|, the minimum. **Perpendicular** (θ = 90°): R = $\sqrt{(A^2 + B^2)}$.

SUBTRACTION: A - B is just A + (-B)

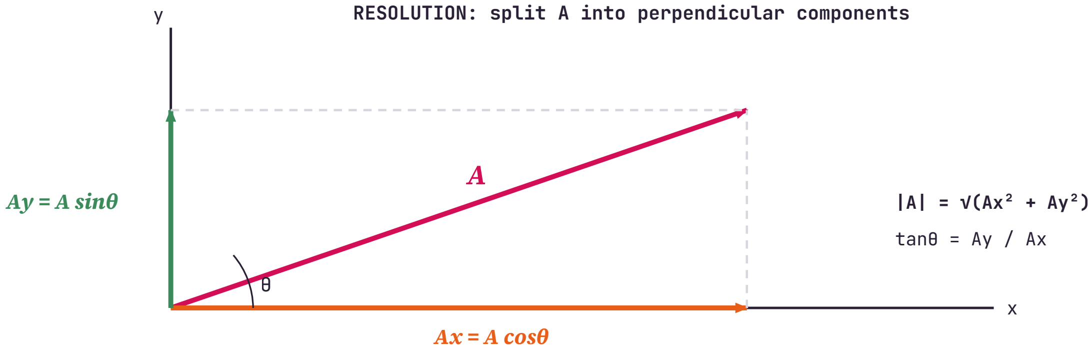
reverse B, then add head-to-tail



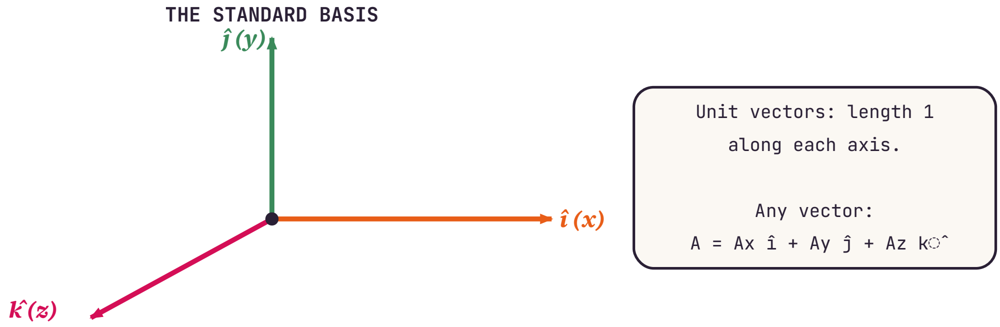
Subtraction is addition in disguise: A minus B means reverse B, then add it head-to-tail. For many vectors, chain them head-to-tail (the polygon law) and the resultant closes the figure.

3 Components and Unit Vectors

The real power of vectors comes from breaking them into perpendicular pieces. Along the axes, addition becomes simple number addition.



Resolving a vector: drop perpendiculars to the axes. The horizontal piece is A cosθ, the vertical piece A sinθ. Together they rebuild A.



The standard basis. Every vector in space is a combination of the unit vectors along x, y and z.

★ COMPONENT FORM

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

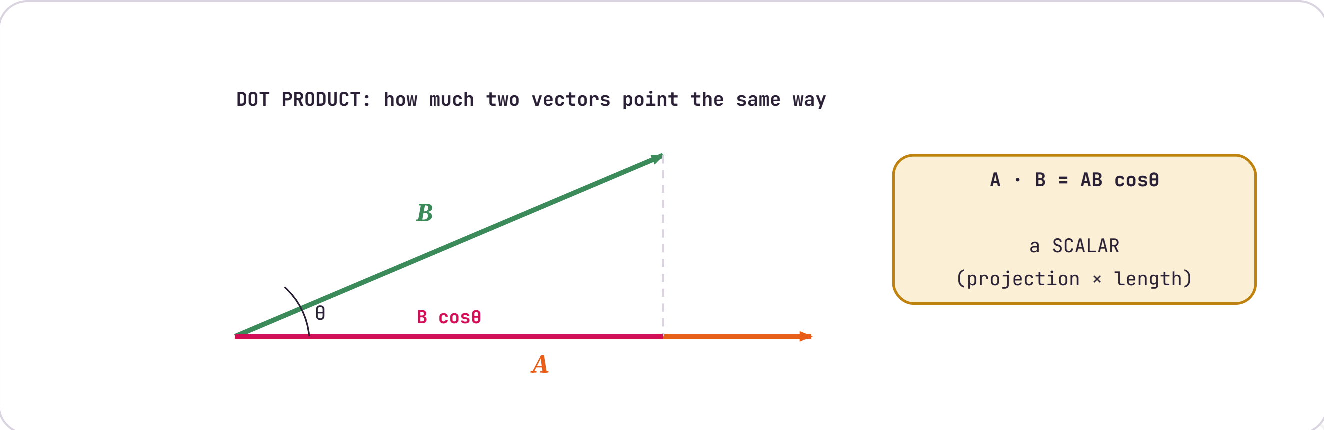
$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}, \qquad \tan\theta = \frac{A_y}{A_x}$$

WHY COMPONENTS MATTER

To add vectors, add their components separately: the x-parts add, the y-parts add. This turns a geometry problem into arithmetic, and it is how every 2D problem in this book is actually solved.

4 The Dot Product

There are two ways to multiply vectors. The **dot product** answers: how much do these two vectors point the same way? Its result is a plain number.



The dot product multiplies one vector by the projection of the other onto it. Maximum when aligned, zero when perpendicular.

★ SCALAR (DOT) PRODUCT

$$\vec{A} \cdot \vec{B} = AB\cos\theta = A_x B_x + A_y B_y + A_z B_z$$

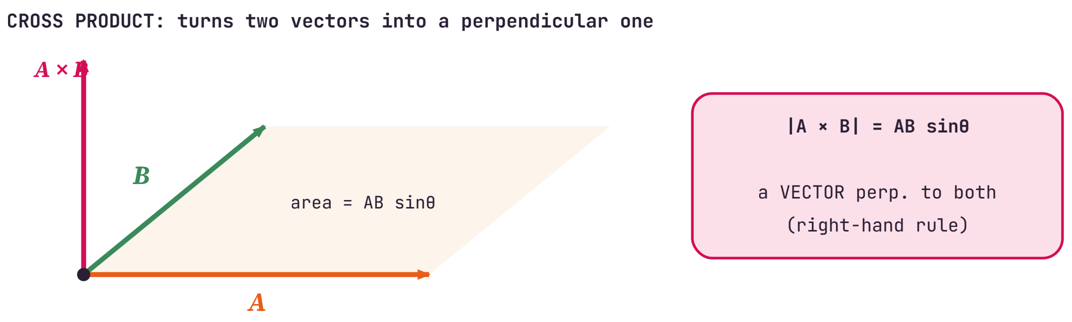
The result is a SCALAR. Use it to find angles and projections.

WHAT IT IS GOOD FOR

Angle between vectors: $\cos\theta = (A\cdot B)/(AB)$. **Test for perpendicular:** $A\cdot B = 0$. **Work done:** $W = F\cdot d$. Note $A\cdot A = A^2$, and $\hat{\imath}\cdot\hat{\imath} = 1$ but $\hat{\imath}\cdot\hat{\jmath} = 0$.

5 The Cross Product

The **cross product** answers a different question: give me a new vector perpendicular to both of these. Its result is a vector, and its size measures the area they span.



The cross product returns a vector at right angles to both inputs (right-hand rule), with magnitude equal to the area of the parallelogram they form.

★ **VECTOR (CROSS) PRODUCT**

$$|\vec{A} \times \vec{B}| = AB\sin\theta$$

The result is a VECTOR, perpendicular to both, pointed by the right-hand rule.

⚠ **DOT VS CROSS, DO NOT MIX THEM**

Dot gives a scalar and is maximum when parallel (cosθ). **Cross** gives a vector and is maximum when perpendicular (sinθ). Parallel vectors have cross product zero; perpendicular vectors have dot product zero. Torque and angular momentum use cross; work and power use dot.

6 Important Questions: Vectors

Worked in the Core, Working, Trap style. Try each before reading on.

Q1 Resultant of two forces

JEE MAIN

Two forces of 3 N and 4 N act at an angle of 60° to each other. Find the magnitude and direction of their resultant.

CORE Parallelogram law: $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$.

WORKING

$$R = \sqrt{3^2 + 4^2 + 2(3)(4)\cos 60^\circ} = \sqrt{25 + 12} = \mathbf{6.08}\ \mathrm{N}$$

$$\tan\alpha = \frac{4\sin 60^\circ}{3 + 4\cos 60^\circ} = \frac{3.46}{5} \Rightarrow \alpha \approx \mathbf{34.7^\circ}\ \text{from the 3 N force}$$

TRAP The 60° is the angle between the forces, not between a force and the resultant. Always state which vector your direction angle is measured from.

Q2 Unit vector along a direction

JEE MAIN

Find the unit vector along $\vec{A} = 3\hat{i} + 4\hat{j}$.

CORE A unit vector is $\hat{A} = \vec{A}/|\vec{A}|$.

WORKING

$$|\vec{A}| = \sqrt{3^2 + 4^2} = 5 \Rightarrow \hat{A} = \frac{3\hat{i} + 4\hat{j}}{5} = \mathbf{0.6\hat{i} + 0.8\hat{j}}$$

TRAP A unit vector must have magnitude 1. Check: $0.6^2 + 0.8^2 = 1$. If it does not, you divided by the wrong number.

Q3 Dot product and angle

JEE MAIN

For $\vec{A}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{B}=2\hat{i}-\hat{j}+\hat{k}$, find $A\cdot B$ and the angle between them.

CORE $\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z$, then $\cos\theta=\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$.

WORKING

$$\vec{A}\cdot\vec{B}=(1)(2)+(2)(-1)+(3)(1)=\mathbf{3}$$

$$\cos\theta=\frac{3}{\sqrt{14}\sqrt{6}}=0.327\Rightarrow\theta\approx\mathbf{70.9}^\circ$$

TRAP The dot product is a scalar, a single number. Do not carry unit vectors into the answer, and remember $\hat{i}\cdot\hat{j}=0$.

Q4 Equal vectors, equal resultant

JEE ADVANCED

Two vectors of equal magnitude a have a resultant of magnitude a. Find the angle between them.

CORE Use $R^2=A^2+B^2+2AB\cos\theta$ with $A=B=a$ and $R=a$.

WORKING

$$a^2=a^2+a^2+2a^2\cos\theta\Rightarrow\cos\theta=-\tfrac{1}{2}\Rightarrow\theta=\mathbf{120}^\circ$$

TRAP The instinctive answer 60° is wrong. Solve for cosθ; a negative cosine means an obtuse angle.

Q5 When |A+B| = |A-B|

JEE ADVANCED

For what angle between A and B is $|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$?

CORE Square both sides: $|\vec{A}\pm\vec{B}|^2=A^2+B^2\pm2\vec{A}\cdot\vec{B}$.

WORKING

$$A^2+B^2+2\vec{A}\cdot\vec{B}=A^2+B^2-2\vec{A}\cdot\vec{B}\Rightarrow\vec{A}\cdot\vec{B}=0\Rightarrow\theta=\mathbf{90}^\circ$$

TRAP This is the classic test for perpendicular vectors: their sum and difference are equal in length only when they are at right angles (the diagonals of a rectangle).

Q6 Find the perpendicular condition

JEE ADVANCED

Find λ so that $\vec{A} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ is perpendicular to $\vec{B} = \hat{i} - 2\hat{j} + 3\hat{k}$.

CORE Perpendicular means $\vec{A} \cdot \vec{B} = 0$.

WORKING

$$(2)(1) + (\lambda)(-2) + (1)(3) = 0 \Rightarrow 5 - 2\lambda = 0 \Rightarrow \lambda = \mathbf{2.5}$$

TRAP Perpendicular uses the dot product equal to zero, not the cross product. Parallel would use the cross product equal to zero.

Q7 Area of a triangle from vectors

JEE ADVANCED

Find the area of the triangle whose two sides are $\vec{A} = \hat{i} + 2\hat{j}$ and $\vec{B} = 3\hat{i} + \hat{j}$.

CORE Triangle area is $\frac{1}{2}|\vec{A} \times \vec{B}|$.

WORKING

$$\vec{A} \times \vec{B} = (1 \cdot 1 - 2 \cdot 3)\hat{k} = -5\hat{k} \Rightarrow |\vec{A} \times \vec{B}| = 5$$

$$\text{Area} = \tfrac{1}{2}(5) = \mathbf{2.5}\ \text{units}^{\mathbf{2}}$$

TRAP The cross product gives the parallelogram area; the triangle is half of it. Forgetting the factor of ½ is the usual slip.

PART B

Kinematics

A rapid, formula-first recap, then the questions that matter.

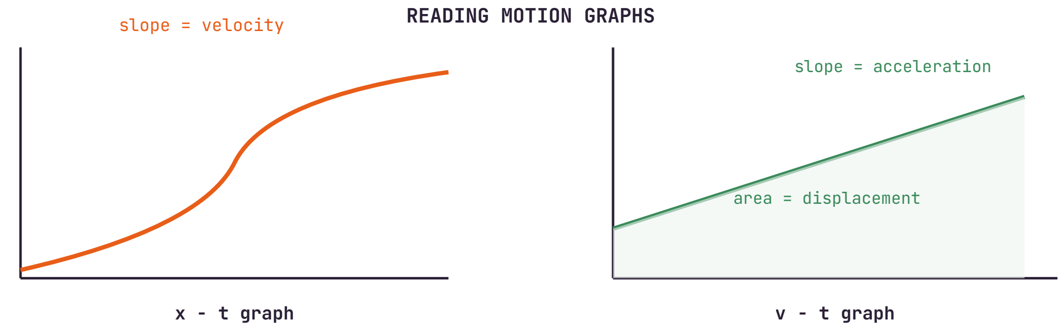
7 Kinematics Recap: 1D

Motion in a straight line, compressed to what you actually use. **Distance** and **speed** are scalars; **displacement** and **velocity** are vectors. Distance ≥ |displacement|, always.

★ EQUATIONS OF UNIFORMLY ACCELERATED MOTION

$$v = u + at \qquad s = ut + \tfrac{1}{2}at^2 \qquad v^2 = u^2 + 2as$$

$$s_n = u + \tfrac{a}{2}(2n - 1) \quad \text{(distance in the } n\text{th second)}$$



The two graphs you must read instantly: on an x-t graph the slope is velocity; on a v-t graph the slope is acceleration and the area is displacement.

FREE FALL

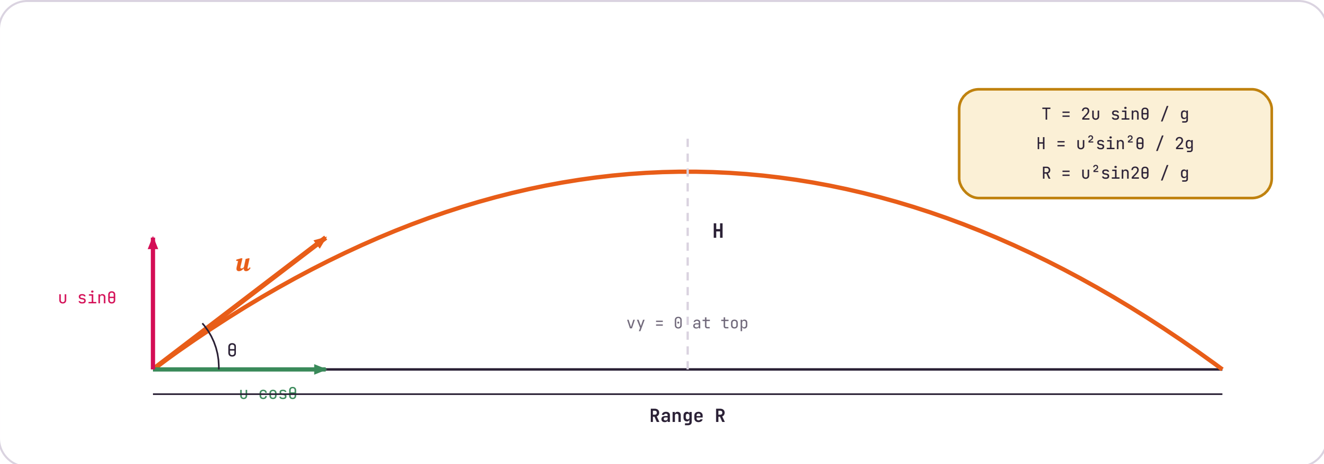
Replace a with g (about 9.8 or 10 m/s^2), downward. Thrown up with speed u: time to top = u/g, total time = 2u/g, maximum height = u^2/2g. At the top velocity is zero but acceleration is still g.

⚠ TWO CLASSIC 1D TRAPS

Zero velocity does not mean zero acceleration (top of a throw). And s_n is the distance **in** the nth second, not the total distance in n seconds.

8 Kinematics Recap: 2D

In two dimensions the trick never changes: split the motion into independent x and y parts. Projectile motion is just horizontal uniform velocity plus vertical free fall.

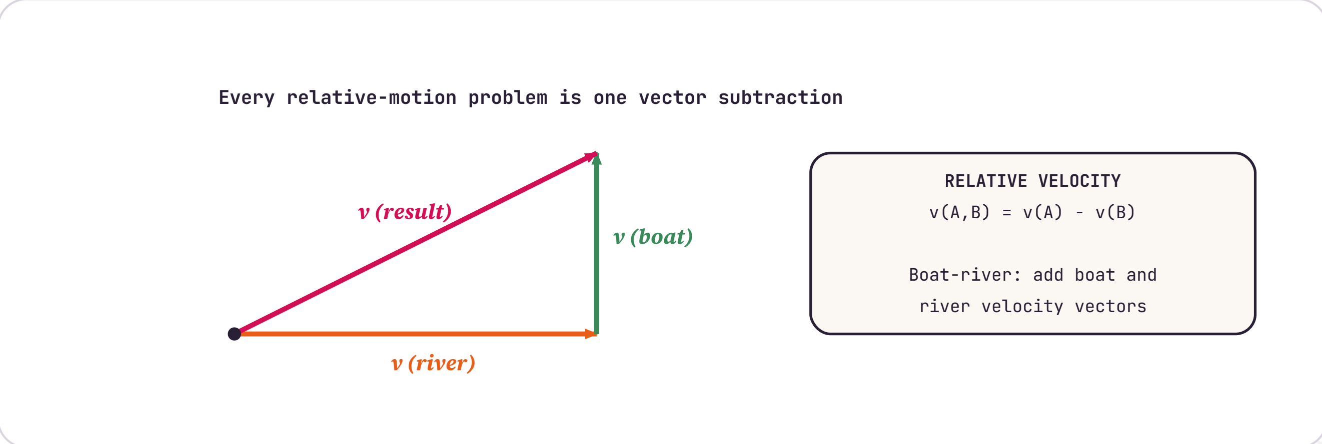


Projectile motion: constant horizontal velocity, vertical free fall. The two are independent and share only the time t.

★ **PROJECTILE (LAUNCH SPEED U AT ANGLE θ)**

$$T = \frac{2u\sin\theta}{g} \qquad H = \frac{u^2\sin^2\theta}{2g} \qquad R = \frac{u^2\sin 2\theta}{g}$$

Range is maximum at θ = 45°. Angles θ and (90° − θ) give the same range.



Relative velocity is one vector subtraction. Boat-and-river, rain-and-man, and pursuit problems are all the same idea in different clothes.

★ **RELATIVE VELOCITY**

$$\vec{v}_{A\,\mathrm{rel}\,B} = \vec{v}_A - \vec{v}_B$$

River crossing: time depends only on the across-component of the boat speed.

9 Important Questions: Kinematics

A heavy set across 1D, projectile and relative motion. Core, Working, Trap.

Q1 Vertical throw

JEE MAIN

A ball is thrown straight up at 20 m/s ($g = 10$). Find the maximum height and the total time to return.

CORE At the top $v = 0$, so $H = u^2/2g$ and total time $= 2u/g$.

WORKING

$$H = \frac{20^2}{2(10)} = \mathbf{20}\ \text{m} \qquad T = \frac{2(20)}{10} = \mathbf{4}\ \text{s}$$

TRAP Time to the top is u/g = 2 s; the total flight is double that. Do not confuse the two.

Q2 Distance in the nth second

JEE MAIN

A body starts from rest with acceleration 10 m/s^2. How far does it travel during the 3rd second?

CORE Use $s_n = u + \frac{a}{2}(2n - 1)$ with $u = 0,\ n = 3$.

WORKING

$$s_3 = 0 + \frac{10}{2}(2 \cdot 3 - 1) = 5(5) = \mathbf{25}\ \text{m}$$

TRAP This is the distance covered only within the 3rd second, not the total distance after 3 seconds (which would be ½·10·9 = 45 m).

Q3 Reading a v-t graph

JEE MAIN

A body's velocity increases uniformly from 0 to 20 m/s in 5 s. Find the distance covered.

CORE On a v-t graph, displacement is the area under the line.

WORKING

$$s = \text{area} = \tfrac{1}{2} \times 5 \times 20 = \mathbf{50}\ \text{m}$$

TRAP You cannot use distance = velocity × time here, because the velocity is not constant. Always take the area.

Q4 Projectile essentials

JEE MAIN

A projectile is launched at 20 m/s at 30° above the horizontal $(g = 10)$. Find its time of flight, maximum height, and range.

CORE $T = \frac{2u\sin\theta}{g},\ H = \frac{u^2\sin^2\theta}{2g},\ R = \frac{u^2\sin 2\theta}{g}.$

WORKING

$$T = \frac{2(20)(0.5)}{10} = \mathbf{2}\ \text{s} \qquad H = \frac{400(0.25)}{20} = \mathbf{5}\ \text{m}$$

$$R = \frac{400\sin 60^\circ}{10} = \frac{400(0.866)}{10} = \mathbf{34.6}\ \text{m}$$

TRAP Height uses sinθ (the vertical part); range uses sin2θ. Mixing the two is the single most common projectile error.

Q5 Rowing straight across a river

JEE MAIN

A boat rows at 5 m/s aimed straight across a 100 m wide river that flows at 3 m/s. Find the time to cross and the drift downstream.

CORE The crossing is done by the across-component (5 m/s); the river only carries the boat sideways.

WORKING

$$t = \frac{100}{5} = \mathbf{20}\ \text{s} \qquad \text{drift} = 3 \times 20 = \mathbf{60}\ \text{m}$$

TRAP Crossing time depends only on the boat's across-speed, never on the river speed. The river sets the drift, not the time.

Q6 Two angles, one range

JEE ADVANCED

A ball thrown at 30° and the same ball thrown at 60° (same speed) land at the same spot. Find the ratio of their maximum heights.

CORE Range depends on $\sin 2\theta$, equal for θ and 90°−θ. Height depends on $\sin^2\theta$.

WORKING

$$\frac{H_{30}}{H_{60}} = \frac{\sin^2 30^\circ}{\sin^2 60^\circ} = \frac{1/4}{3/4} = \mathbf{\frac{1}{3}}$$

TRAP Equal range does not mean equal height. Complementary angles share a range but the steeper one flies far higher.

Q7 Crossing with zero drift

JEE ADVANCED

The same boat (5 m/s) must cross the 100 m river (flow 3 m/s) landing exactly opposite its start. At what angle must it head, and how long does it take?

CORE Head upstream so the upstream component cancels the river: $v_b \sin\phi = v_r$.

WORKING

$$\sin\phi = \frac{3}{5} = 0.6 \Rightarrow \phi = \mathbf{36.9^\circ} \text{ upstream}$$

$$v_{\text{across}} = 5\cos\phi = 4 \text{ m/s} \Rightarrow t = \frac{100}{4} = \mathbf{25} \text{ s}$$

TRAP Zero drift costs time: 25 s here versus 20 s for the straight-across route. Minimum-time and minimum-drift are different paths.

Q8 The umbrella angle

JEE ADVANCED

Rain falls vertically at 4 m/s. A man walks at 3 m/s on level ground. At what angle from the vertical should he tilt his umbrella?

CORE Hold the umbrella along the rain's velocity relative to the man, $\vec{v}_{\text{rain}} - \vec{v}_{\text{man}}$.

WORKING

$$\tan\theta = \frac{v_{\text{man}}}{v_{\text{rain}}} = \frac{3}{4} \Rightarrow \theta = \mathbf{36.9^\circ} \text{ from vertical}$$

TRAP Tilt the umbrella forward, into the direction of walking. Relative to the man the rain appears to come from ahead, not from straight above.

Q9 Horizontal projectile from a cliff

JEE ADVANCED

A stone is thrown horizontally at 15 m/s from a 20 m high cliff $(g = 10)$. Find the time to land, the horizontal range, and the speed on landing.

CORE Vertical is free fall from rest $(u_y = 0)$; horizontal is uniform. They share only the time.

WORKING

$$t = \sqrt{\frac{2h}{g}} = \sqrt{4} = 2\text{ s} \qquad x = 15 \times 2 = \mathbf{30}\text{ m}$$

$$v_y = gt = 20, \quad v = \sqrt{15^2 + 20^2} = \mathbf{25}\text{ m/s}$$

TRAP The initial vertical velocity is zero, so the fall time depends only on the height, not on the launch speed. Horizontal and vertical motions never talk to each other except through t.

Q10 Velocity given as a function of position

JEE ADVANCED

A particle moves so that its velocity is $v = 2\sqrt{x}$ (SI units). Find its acceleration.

CORE When v depends on position, use $a = v\frac{dv}{dx}$.

WORKING

$$\frac{dv}{dx} = \frac{1}{\sqrt{x}} \Rightarrow a = v\frac{dv}{dx} = 2\sqrt{x} \cdot \frac{1}{\sqrt{x}} = \mathbf{2}\text{ m/s}^{\mathbf{2}}$$

TRAP You cannot write a = dv/dt directly here. Use $a = v\,dv/dx$; the surprising result is a constant acceleration.

★ Vectors & Kinematics · Fact Sheet

Every formula for revision day. Print this page alone.

VECTOR ADDITION

$R = \sqrt{(A^2+B^2+2AB\cos\theta)}$
$\tan\alpha = B\sin\theta/(A+B\cos\theta)$
max A+B, min |A-B|.

COMPONENTS

$A = Ax\,\hat{i} + Ay\,\hat{j} + Az\,\hat{k}$
$Ax = A\cos\theta$, $Ay = A\sin\theta$
$|A| = \sqrt{(Ax^2+Ay^2)}$.

DOT PRODUCT

$A\cdot B = AB\cos\theta$ (scalar).
$= AxBx+AyBy+AzBz$.
Perp: $A\cdot B = 0$.

CROSS PRODUCT

$|A\times B| = AB\sin\theta$ (vector).
perp. to both, right-hand rule.
Parallel: $A\times B = 0$.

1D MOTION

$v = u+at$
$s = ut+½at^2$
$v^2 = u^2+2as$.

GRAPHS & NTH SEC

x-t slope = v; v-t slope = a.
v-t area = displacement.
$s_n = u + (a/2)(2n-1)$.

PROJECTILE

$T = 2u\sin\theta/g$
$H = u^2\sin^2\theta/2g$
$R = u^2\sin2\theta/g$ (max at 45°).

RELATIVE VELOCITY

v(A rel B) = v(A) - v(B).
River time = width/across-speed.
Zero drift: $\sin\phi = vr/vb$.

FREE FALL

up-speed u: top at u/g,
flight 2u/g, height $u^2/2g$.
At top v=0 but a=g.