LOGIC BLOOM · JEE NOTES

Sets

Chapter 1 · *Class 11 Mathematics*

Complete Notes for JEE Main & Advanced

Theory · Formulas · Solved Examples · Traps · Practice

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Understand through games.
Score through practice.

Contents

PART I

Theory & Formulas

1 What is a Set?

A set is a well-defined collection of distinct objects. Well-defined means that given any object, we can decide unambiguously whether it belongs to the collection. The objects are called elements or members.

If a is an element of set A, we write $a \in A$ (read "a belongs to A"). If not, we write $a \notin A$. Sets are denoted by capital letters; elements by lowercase letters. Collections that are **not** sets — "all tall students", "the best cricketers" — are not well-defined.

⚠ TRAP

Order and repetition do not matter: $\{1, 2, 3\} = \{3, 2, 1\} = \{1, 1, 2, 3\}$. A set never counts duplicates.

2 Representation of Sets

Form	Description	Example
Roster / Tabular	List all elements within braces, separated by commas.	$A = \{2, 4, 6, 8\}$
Set-builder	State the defining property of the elements.	$A = \{x : x = 2n, n \in N, n \leq 4\}$

3 Types of Sets

Type	Definition	Example / Symbol
Empty (Null / Void)	Set with no elements.	$\varnothing$ or { }
Singleton	Set with exactly one element.	{0}, {$\varnothing$}
Finite	Countable number of distinct elements.	{a, e, i, o, u}
Infinite	Elements cannot be counted or listed fully.	N, Z, points on a line
Equal	Same elements exactly: $A = B$.	$\{1,2,3\} = \{3,1,2\}$
Equivalent	Same cardinality: $n(A) = n(B)$.	$\{a,b,c\} \leftrightarrow \{1,2,3\}$
Subset	Every element of A is in B: $A \subseteq B$.	$N \subseteq Z \subseteq Q \subseteq R$
Proper subset	$A \subseteq B$ and $A \neq B$: $A \subsetneq B$.	$\{1,2\} \subsetneq \{1,2,3\}$
Universal	Set containing all objects under discussion.	U
Power set	Set of all subsets of A: P(A).	see §5

KEY FACTS — EMPTY SET & SUBSETS

$\varnothing$ is a subset of every set, and $\varnothing \subseteq \varnothing$. Every set is a subset of itself: $A \subseteq A$.

$\varnothing \neq \{0\} \neq \{\varnothing\}$ — the empty set, a set containing zero, and a set containing the empty set are all different.

4 Intervals as Subsets of R

Interval	Set-builder	Includes endpoints?
(a, b)	$\{x : a < x < b\}$	Neither (open)
$[a, b]$	$\{x : a \le x \le b\}$	Both (closed)
$[a, b)$	$\{x : a \le x < b\}$	Left only
$(a, b]$	$\{x : a < x \le b\}$	Right only

Length of each interval = $b - a$. The number of elements is infinite for any non-degenerate real interval.

5 Power Set & Cardinality

CORE COUNTING FORMULAS — IF N(A) = M

Number of subsets of A = 2^m · Number of proper subsets = $2^m - 1$

Number of non-empty subsets = $2^m - 1$ · Number of non-empty proper subsets = $2^m - 2$

$n(P(A)) = 2^m$ · $n(P(P(A))) = 2^{(2^m)}$

6 Operations on Sets

Operation	Definition	Symbol / Notation
Union	Elements in A or B (or both).	$A \cup B = \{x : x \in A \text{ or } x \in B\}$
Intersection	Elements in both A and B.	$A \cap B = \{x : x \in A \text{ and } x \in B\}$
Difference	In A but not in B.	$A - B = \{x : x \in A, x \notin B\}$
Sym. difference	In exactly one of A, B.	$A \Delta B = (A-B) \cup (B-A)$
Complement	In U but not in A.	$A' = U - A$
Disjoint	No common elements.	$A \cap B = \varnothing$

7 Algebra of Sets (Laws)

Identity: $A \cup \varnothing = A$ · $A \cap U = A$ · $A \cup U = U$ · $A \cap \varnothing = \varnothing$
Idempotent / Complement: $A \cup A = A$ · $A \cap A = A$ · $A \cup A' = U$ · $A \cap A' = \varnothing$ · $(A')' = A$
Commutative: $A \cup B = B \cup A$ · $A \cap B = B \cap A$
Associative: $(A \cup B) \cup C = A \cup (B \cup C)$ · $(A \cap B) \cap C = A \cap (B \cap C)$
Distributive: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

DE MORGAN'S LAWS — (MOST TESTED) ★

$(A \cup B)' = A' \cap B'$ · $(A \cap B)' = A' \cup B'$

⚠ TRAP

$A - B = A \cap B'$. Also $A - B$, $B - A$, and $A \cap B$ are mutually disjoint and partition $A \cup B$.

8 Cardinality Formulas (Inclusion–Exclusion)

Two sets

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B) = n(A-B) + n(B-A) + n(A\cap B)$

$n(A - B) = n(A) - n(A \cap B) = n(\text{only } A)$

Three sets

$n(A \cup B \cup C) = n(A) + n(B) + n(C)$

$\quad - n(A\cap B) - n(B\cap C) - n(C\cap A) + n(A\cap B\cap C)$

Complement & difference counts

$n(A') = n(U) - n(A)$ · $n(A' \cap B') = n((A\cup B)') = n(U) - n(A\cup B)$

$n(\text{exactly two}) = n(A\cap B) + n(B\cap C) + n(C\cap A) - 3\cdot n(A\cap B\cap C)$

$n(\text{exactly one}) = \Sigma n(A) - 2\Sigma n(A\cap B) + 3n(A\cap B\cap C)$

PART II

Solved Examples

JEE MAIN

Q1. If $A = \{x : x \text{ is a multiple of } 2\}$, $B = \{x : x \text{ is a multiple of } 5\}$, $C = \{x : x \text{ is a multiple of } 10\}$, all over N, then $A \cap (B \cup C)$ equals?

Every multiple of 10 is already a multiple of 5, so $C \subseteq B$ and $B \cup C = B$. Then $A \cap B$ = numbers that are multiples of both 2 and 5 = multiples of 10 = C. **Answer: C.**

JEE MAIN

Q2. If a set A has 4 elements, how many subsets of A contain at least one element?

Total subsets = $2^4 = 16$. Remove the empty set: $16 - 1 = 15$. **Answer: 15.**

JEE MAIN

Q3. In a group of 100 people, 65 like cricket, 40 like tennis. How many like both, given everyone likes at least one?

$n(C \cup T) = 100$, $n(C) = 65$, $n(T) = 40$. By inclusion–exclusion: $100 = 65 + 40 - n(C \cap T) \Rightarrow n(C \cap T) = 5$. **Answer: 5.**

JEE ADVANCED

Q4. Let $X = \{1, 2, 3, \ldots, 2017\}$. Let S be the set of all subsets of X whose elements sum to an even number. Find n(S).

X has 1009 odd numbers and 1008 even numbers. A subset's sum is even iff it contains an even count of odd numbers. Choosing any subset of the 1008 evens (2^{1008} ways) and an even-sized subset of the 1009 odds (2^{1008} ways, since exactly half of 2^{1009} subsets have even size) gives $n(S) = 2^{1008} \times 2^{1008} =$ $\mathbf{2^{2016}}$. *Key idea:* among subsets of an n-element set, exactly 2^{n-1} have even size (for $n \geq 1$).

JEE ADVANCED

Q5. For sets A and B, define $A \Delta B = (A-B) \cup (B-A)$. Prove that $A \Delta B = \varnothing$ iff $A = B$.

($\Rightarrow$) If $A \Delta B = \varnothing$, then $A-B = \varnothing$ and $B-A = \varnothing$. $A-B = \varnothing$ means $A \subseteq B$; $B-A = \varnothing$ means $B \subseteq A$. Hence $A = B$.
($\Leftarrow$) If $A = B$, then $A-B = \varnothing$ and $B-A = \varnothing$, so their union $A \Delta B = \varnothing$. □

JEE ADVANCED

Q6. In a survey of 600 students: 150 take tea, 225 take coffee, 100 take both. Additionally 300 take milk, 75 take tea+milk, 80 take coffee+milk, 50 take all three. How many take none of the three?

Three-set inclusion–exclusion: $n(T \cup C \cup M) = 150 + 225 + 300 - 100 - 80 - 75 + 50 = 470$. Students taking none $= 600 - 470 =$ **130.**

PART III

Common Traps & PYQ Patterns

⚠ TRAP 1 — ∅ VS {∅} VS {0}

These three are pairwise distinct. $n(\varnothing) = 0$, $n(\{\varnothing\}) = 1$, $n(\{0\}) = 1$. The set $\{\varnothing\}$ has one element (which is itself a set). Examiners love asking whether $\varnothing \in \{\varnothing\}$ (true) vs $\varnothing \subseteq \{\varnothing\}$ (also true).

⚠ TRAP 2 — SUBSET (⊆) VS ELEMENT (∈)

For $A = \{1, \{2,3\}, 4\}$: here $1 \in A$ is true, but $\{1\} \subseteq A$ is also true. However $\{2,3\} \in A$ is true while $2 \in A$ is false (2 is an element of an element, not of A). Mixing $\in$ and $\subseteq$ is the single most common error.

⚠ TRAP 3 — "AT LEAST / AT MOST / EXACTLY"

In Venn problems, distinguish carefully: exactly one region vs at least one (= union). n(at least two) $= n(A\cap B) + n(B\cap C) + n(C\cap A) - 2n(A\cap B\cap C)$. The pairwise counts $n(A\cap B)$ usually include the triple region — subtract it when you want "exactly two".

⚠ TRAP 4 — POWER SET COUNTING

"Number of proper subsets" $= 2^m - 1$ (excludes A itself, includes ∅). But some problems define proper subset to exclude both A and ∅ — read the wording. $n(P(P(A)))$ grows as $2^{(2^m)}$, not 2^{2m}.

⚠ TRAP 5 — DE MORGAN SIGN FLIP

$(A \cup B)' = A' \cap B'$ (union flips to intersection). Students routinely write $A' \cup B'$, which is wrong. The operation inverts under complement.

▲ ADVANCED PATTERN — MAXIMISING / MINIMISING OVERLAP

Given $n(A)$, $n(B)$, $n(U)$: $n(A\cap B)$ is maximum $= \min(n(A), n(B))$; it is minimum $= \max(0, n(A)+n(B) - n(U))$. These bounds appear in JEE Advanced optimisation-style set questions.

▲ ADVANCED PATTERN — FUNCTIONAL / IDENTITY PROOFS

Proving identities like $A - (B \cap C) = (A-B) \cup (A-C)$ is best done by the element-membership method: show $x \in \text{LHS} \Leftrightarrow x \in \text{RHS}$. This rigorous two-way argument is expected in Advanced subjective-style reasoning.

PART IV

Practice Set

SECTION A — JEE MAIN LEVEL

1. If $n(A) = 3$ and $n(B) = 6$ with $A \subseteq B$, find the minimum and maximum value of $n(A \cup B)$.
2. Write the set $\{x : x \text{ is an integer}, x^2 \leq 9\}$ in roster form.
3. If $A = \{1, 2, \{3,4\}, 5\}$, state whether each is true: (i) $\{3,4\} \subseteq A$, (ii) $\{3,4\} \in A$, (iii) $1 \in A$, (iv) $\{1\} \in A$.
4. A set has 255 proper subsets (excluding the set itself). How many elements does it have?
5. In a class of 35 students, 24 like maths, 16 like physics, and 8 like both. How many like neither?
6. If $A = \{x : x = 3n, n \in N, n \leq 4\}$ and $B = \{x : x = 2n, n \in N, n \leq 6\}$, find $A \cap B$ and $A \Delta B$.
7. Prove using laws: $A \cup (A \cap B) = A$ (absorption law).

SECTION B — BORDERLINE (MAIN / ADVANCED)

8. Of 200 students, 120 study Hindi, 90 study English, 40 study French. 35 study Hindi+English, 20 English+French, 15 Hindi+French, 10 all three. Find how many study (a) only Hindi, (b) exactly one language, (c) none.
9. If $n(U) = 50$, $n(A) = 28$, $n(B) = 25$, find the greatest and least possible value of $n(A \cap B)$.
10. Let A and B be two sets such that $n(A) = 20$, $n(A \cup B) = 42$, $n(A \cap B) = 4$. Find $n(B)$ and $n(B - A)$.
11. Prove: $A - (B \cup C) = (A - B) \cap (A - C)$.

SECTION C — JEE ADVANCED LEVEL

12. Let $S = \{1, 2, 3, \ldots, n\}$. Find the number of unordered pairs (A, B) of disjoint non-empty subsets of S.
13. For $X = \{1,2,3,\ldots,10\}$, find the number of subsets whose elements sum to an odd number.
14. If $A_1, A_2, \ldots, A_{30}$ are sets each with 5 elements, and every element belongs to exactly 10 of these sets, while the union has m elements, find m.
15. Prove that for any three sets, $n(A \cup B \cup C)$ is minimised when overlaps are maximised, and state the minimum given $n(A)=n(B)=n(C)=k$ inside a universe of size $2k$.

KEY

Answer Key

Q	Answer
1	Since $A \subseteq B$, $A \cup B = B$, so $n(A \cup B) = 6$ (both min and max).
2	$\{-3, -2, -1, 0, 1, 2, 3\}$.
3	(i) False — $\{3,4\}$ is an element, not a subset; (ii) True; (iii) True; (iv) False ($\{1\} \subseteq A$ is true, but $\{1\} \in A$ is false).
4	$2^m - 1 = 255 \Rightarrow 2^m = 256 \Rightarrow m = 8$.
5	$n(M \cup P) = 24 + 16 - 8 = 32$; neither $= 35 - 32 = 3$.
6	$A = \{3,6,9,12\}$, $B = \{2,4,6,8,10,12\}$. $A \cap B = \{6,12\}$; $A \Delta B = \{2,3,4,8,9,10\}$.
7	$A \cap B \subseteq A$, so $A \cup (A \cap B) = A$ by union with a subset. □
8	(a) only Hindi $= 120 - 35 - 15 + 10 = 80$; (b) exactly one $= (120+90+40) - 2(35+20+15) + 3(10) = 250 - 140 + 30 = 140$; (c) total studying $= 190$, none $= 200 - 190 = 10$.
9	Max $n(A \cap B) = \min(28,25) = 25$; Min $= \max(0, 28+25-50) = 3$.
10	$n(B) = n(A \cup B) + n(A \cap B) - n(A) = 42 + 4 - 20 = 26$; $n(B-A) = 26 - 4 = 22$.
11	Element method: $x \in A-(B \cup C) \Leftrightarrow x \in A, x \notin B, x \notin C \Leftrightarrow (x \in A, x \notin B)$ and $(x \in A, x \notin C) \Leftrightarrow x \in (A-B) \cap (A-C)$. □
12	Each element has 3 choices (A, B, or neither): 3^n. Subtract cases where A or B is empty, then halve for unordered: $(3^n - 2^{n+1} + 1)/2$.
13	$2^9 = 512$ (exactly half of all 2^{10} subsets have an odd sum, since at least one odd number exists).
14	Total element-memberships $= 30 \times 5 = 150$. Each element counted 10 times $\Rightarrow m = 150 / 10 = 15$.
15	Minimum of $n(A \cup B \cup C) = k$ (when $B, C \subseteq A$, all overlap maximally); fits within the universe of size $2k$. Maximising overlap minimises the union.

★ Sets — One-Page Formula Sheet

Everything condensed. Print this page alone.

NOTATION

a ∈ A (belongs) · a ∉ A
A ⊆ B (subset) · A ⊊ B (proper)
∅ = { } empty · U = universal
n(A) = cardinality · P(A) = power set

SUBSET & POWER SET COUNTS

if n(A) = m:
subsets 2^m · proper 2^m-1
non-empty 2^m-1 · n.e. proper 2^m-2
n(P(A)) = 2^m
n(P(P(A))) = $2^{(2^m)}$
even-size subsets 2^{m-1}

OPERATIONS

A∪B = {x : x∈A or x∈B}
A∩B = {x : x∈A and x∈B}
A-B = A ∩ B′
AΔB = (A-B)∪(B-A)
A′ = U - A · disjoint: A∩B=∅

ALGEBRA OF SETS

identity: A∪∅=A · A∩U=A
domination: A∪U=U · A∩∅=∅
idempotent: A∪A=A · A∩A=A
complement: A∪A′=U · A∩A′=∅ · (A′)′=A
absorption: A∪(A∩B)=A
commutative · associative · distributive

DE MORGAN ★

(A ∪ B)′ = A′ ∩ B′
(A ∩ B)′ = A′ ∪ B′
union ⇔ intersection flips under ′

CARDINALITY — TWO SETS

n(A∪B) = n(A)+n(B) - n(A∩B)
n(A-B) = n(A) - n(A∩B)
n(only A) = n(A) - n(A∩B)
n(AΔB) = n(A)+n(B) - 2n(A∩B)
n(A′) = n(U) - n(A)

CARDINALITY — THREE SETS

n(A∪B∪C) = n(A)+n(B)+n(C)
  - n(A∩B) - n(B∩C) - n(C∩A)
  + n(A∩B∩C)
exactly two: Σn(A∩B) - 3n(A∩B∩C)
exactly one: Σn(A) - 2Σn(A∩B) + 3n(∩)
at least two: Σn(A∩B) - 2n(A∩B∩C)

OVERLAP BOUNDS

max n(A∩B) = min(n(A), n(B))
min n(A∩B) = max(0, n(A)+n(B) -n(U))
max overlap ⇒ min union
min overlap ⇒ max union

INTERVALS (SUBSETS OF R)

(a, b): a < x < b
[a, b]: a ≤ x ≤ b
[a, b): a ≤ x < b
(a, b]: a < x ≤ b
length = b - a · N ⊆ Z ⊆ Q ⊆ R

WATCH OUT ⚠

∅ ≠ {0} ≠ {∅}
n(∅)=0, n({∅})=1
∅ ⊆ every set · ∅ ∈ {∅}
∈ (element) ≠ ⊆ (subset)

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