PHYSICS · CLASS 11 · CHAPTER 6 · JEE AND NEET

Rotational Motion

System of particles, torque, moment of inertia and rolling

Everything you learnt about straight-line motion has a twin in rotation. Find the twin, and this chapter stops being new.

14 diagrams | 17 worked questions | JEE Advanced set | Fact sheet

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In short

- The centre of mass moves as if all the mass sat there. All the external forces act as if applied there.
- Torque is to rotation what force is to translation. Moment of inertia plays the role of mass.
- Moment of inertia depends on the axis. The parallel and perpendicular axis theorems move you between axes.
- With no external torque, angular momentum stays constant. Kinetic energy need not.
- Rolling without slipping is translation plus rotation, with the contact point at rest.

0 How to Read This Set

- Keep one idea in mind all through: every linear quantity has a rotational twin. The table in Section 11 lists them.
- Sections 01 to 07 build the tools. Sections 08 to 13 use them on spinning bodies.
- Check each section's badge. Green is for everyone. Orange is for JEE. Magenta is mostly JEE Advanced.
- Orange boxes are traps. Each one comes from a mistake real papers punish.
- Try every worked question before reading its answer. The JEE Advanced set at the end is meant to be hard.

WHICH EXAM NEEDS WHAT

The rationalised NCERT book dropped two sections from this chapter. They are the axes theorems and rolling motion. But the exams did not drop them equally.

- **Axes theorems:** still listed in the NEET 2026 and JEE Main 2026 syllabi. Everyone must learn them (Section 09).
- **Rolling motion:** not listed in the NEET 2026 syllabus. The JEE Main 2026 syllabus does not name it either, but recent JEE Main papers still ask it. The JEE Advanced syllabus lists "rolling without slipping of rings, cylinders and spheres". So Section 13 is for JEE students.

NEET students can skip Section 13 and the JEE Advanced set.

PYQ ANALYSIS **See which mechanics topics JEE Main tests most** →

In our JEE Main analysis, rotational motion was the heaviest mechanics topic. It gave about 1.5 to 2 questions a shift. Moment of inertia alone was 22% of mechanics questions. We have not yet analysed NEET for this chapter.

01 Rigid Bodies and Kinds of Motion

IN NEET AND JEE

- So far, we treated bodies as point particles. Real bodies have size.
- A **rigid body** has a definite shape that does not change.
- The distance between any two of its particles stays fixed.
- No real body is perfectly rigid. But many bodies bend so little that we can treat them as rigid.
- A rigid body can move in three ways:

  1. **Pure translation:** every particle has the same velocity at any instant.
  2. **Rotation about a fixed axis:** every particle moves in a circle around the axis. Each circle lies in a plane perpendicular to the axis, with its centre on the axis.
  3. **Translation plus rotation:** a wheel rolling down a slope does both.

- A body fixed at a point (like a spinning top) can also rotate. Then the axis itself can change direction. We study only the fixed-axis case here.

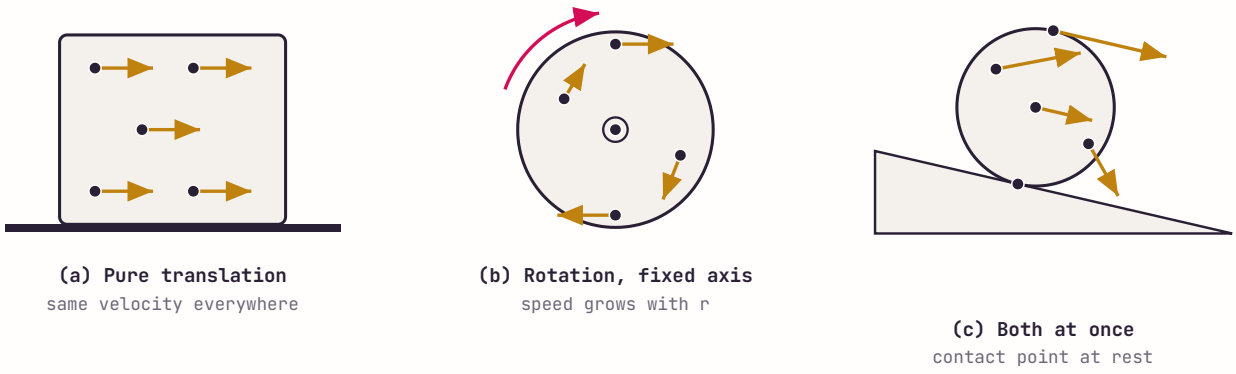


Fig 1 · Three kinds of motion. Translation, rotation, and both at once.



THINK IT THROUGH

SANITY CHECK

- In pure rotation, particles on the axis do not move at all. Particles farther from the axis move faster.

02 Centre of Mass

IN NEET AND JEE | OCCASIONAL | *4% of JEE Main mechanics questions*

Two particles

- The **centre of mass (CM)** is one point that stands for the whole mass when we describe motion.
- For two particles on the x-axis, of masses m_1 and m_2 at x_1 and x_2:

$$X = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

- For equal masses, the CM lies exactly midway.
- For unequal masses, it lies closer to the heavier mass.

Many particles

$$X = \frac{\sum m_i x_i}{M}, \quad Y = \frac{\sum m_i y_i}{M}, \quad Z = \frac{\sum m_i z_i}{M}$$

- Here $M = \sum m_i$ is the total mass.
- In vector form: $\vec{R} = \frac{\sum m_i \vec{r}_i}{M}$.

Continuous bodies

- A solid body has particles packed so close that we treat its mass as continuous.
- Divide it into tiny pieces of mass dm. Replace the sum with an integral:

$$X = \frac{1}{M}\int x\, dm, \quad Y = \frac{1}{M}\int y\, dm, \quad Z = \frac{1}{M}\int z\, dm$$

- For a **uniform body with symmetry**, the CM lies at the geometric centre.
  - Uniform rod: at its midpoint.
  - Ring, disc, sphere, cube: at the centre.
  - Uniform triangular lamina: at the centroid, where the medians meet.
- The CM need not lie inside the material. A ring's CM is at its empty centre.

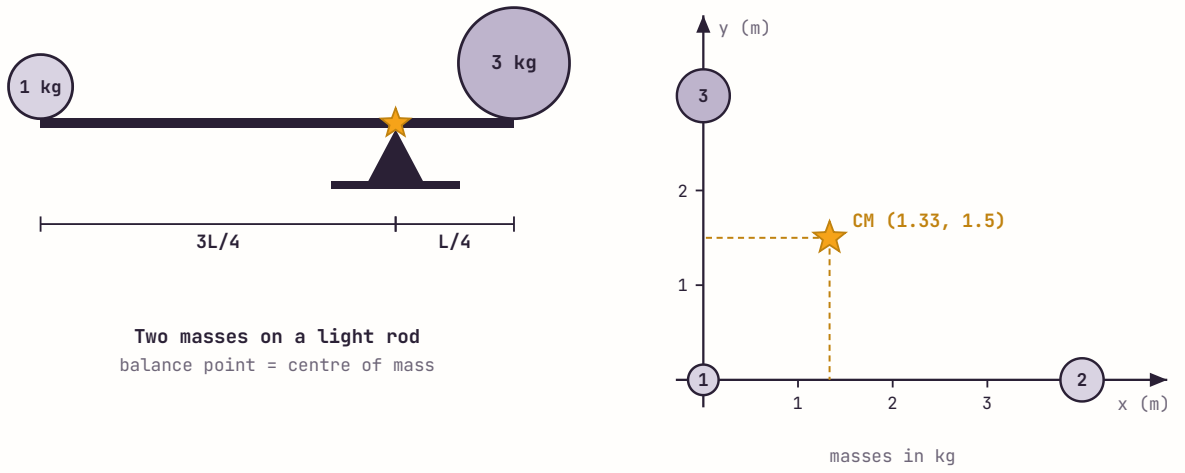


Fig 2 · Centre of mass. The CM sits closer to the heavier mass.

Bodies with a piece cut out (JEE favourite)

- Treat the hole as a body of **negative mass**.
- Find the mass of the removed piece from the area ratio.
- Area scales as radius squared. So a hole of radius $R/2$ in a disc of radius R removes $\frac{1}{4}$ of the mass.

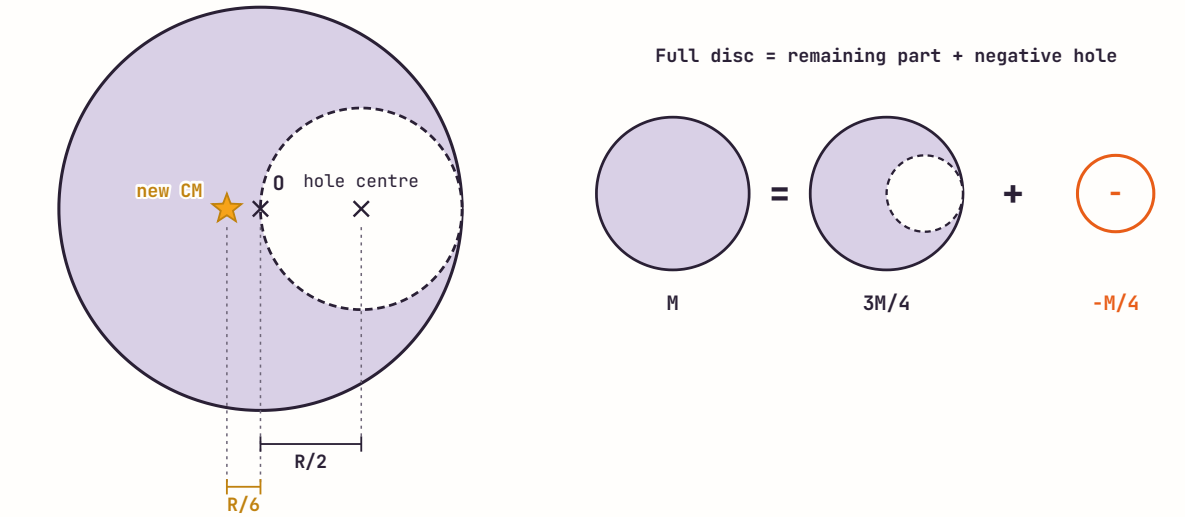


Fig 3 · Disc with a hole. Treat the hole as negative mass: the CM moves R/6 away.



TRAP ALERT

TRAP ALERT

- A hole of half the radius removes one quarter of the mass, not half. This exact slip appears in JEE Main papers.

Q1 Worked question

NEET JEE MAIN LEVEL 2

Masses of 1 kg, 2 kg and 3 kg sit at (0, 0), (4 m, 0) and (0, 3 m). How far is their centre of mass from the origin?

1. 1.7 m
2. 2.0 m
3. 2.8 m
4. 4.0 m

Answer: 2

CORE PRINCIPLE Find each coordinate separately, weighted by mass. Then find the distance.

MATH $X = \frac{2\times4}{6} = 1.33$ m. $Y = \frac{3\times3}{6} = 1.5$ m. Distance $= \sqrt{1.33^2 + 1.5^2} \approx 2.0$ m.

TRAP Option 1 ignores the masses and averages the points. Option 3 adds X and Y instead of combining them as a vector. Option 4 divides by 3, the number of particles, instead of the total mass, 6 kg.

Q2 Worked question

JEE MAIN LEVEL 2

A uniform disc of radius R has a circular hole of radius $R/2$ cut out. The hole's centre is $R/2$ from the disc's centre. How far does the centre of mass shift, and which way?

1. $R/8$, towards the hole
2. $R/2$, away from the hole
3. $R/6$, towards the hole
4. $R/6$, away from the hole

Answer: 4

CORE PRINCIPLE Full disc = remaining part + hole. Put the hole in as negative mass.

MATH Hole mass $= M/4$ at $x = R/2$. Remaining mass $= 3M/4$. Its CM: $X = \frac{0 - (M/4)(R/2)}{3M/4} = -\frac{R}{6}$. The minus sign means away from the hole.

TRAP Option 2 takes the hole's mass as $M/2$. Option 1 forgets to reduce the mass in the denominator. Option 3 gets the size right but loses the minus sign. Removing mass pushes the CM away from the hole.

03 Motion of the Centre of Mass and Linear Momentum

IN NEET AND JEE | OCCASIONAL

The CM moves like a single particle

- Differentiate $M\vec{R} = \sum m_i \vec{r}_i$ twice with time:

$$M\vec{a}_{cm} = \vec{F}_{ext}$$

- Internal forces come in equal and opposite pairs, by Newton's third law. So they cancel in the sum.
- Only **external** forces change the motion of the CM.
- So the CM moves as if all the mass were there, with all the external forces acting there.
- We do not need to know the internal forces to track the CM.

Linear momentum of a system

- Total momentum $\vec{P} = \sum m_i \vec{v}_i = M\vec{V}_{cm}$.
- Newton's second law for a system: $\frac{d\vec{P}}{dt} = \vec{F}_{ext}$.
- If the external force is zero, $\vec{P}$ is constant, so the CM moves with constant velocity.
- **Example:** a radium nucleus at rest decays into a radon nucleus and an alpha particle. The two fly apart in opposite directions. The CM stays at rest.
- **Example:** a shell explodes in mid-air. Gravity is the only external force. So the CM keeps following the same parabola, even after the explosion.

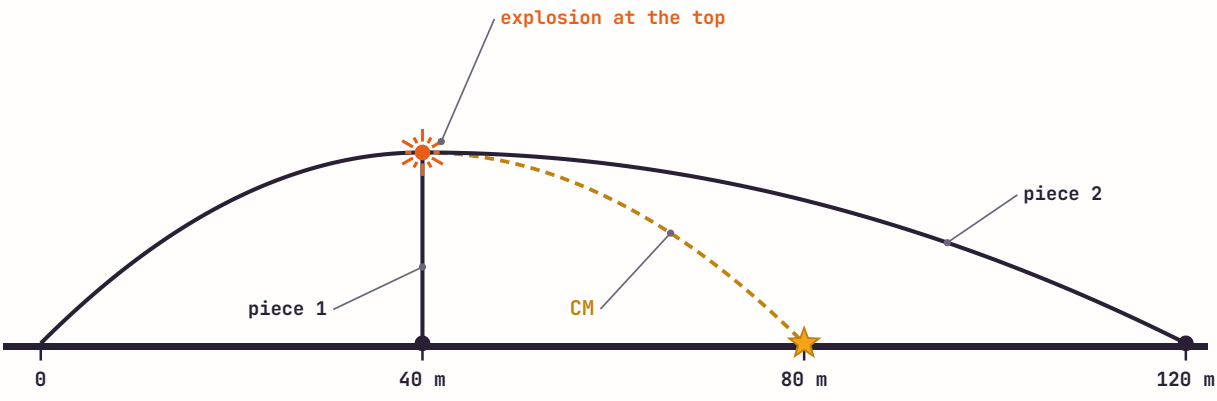


Fig 4 · Exploding shell. The CM keeps its parabola after the explosion.



THINK IT THROUGH

SANITY CHECK

- An explosion changes the pieces' paths, not the CM's path. Only external forces can bend the CM's path.

Q3 Worked question

NEET LEVEL 2

A shell is fired at 45°. Without an explosion, it would land 80 m away on level ground. At the top of its path, it explodes into two equal pieces. One piece falls straight down from rest. Where does the other piece land? Ignore air resistance.

1. 40 m from the gun
2. 80 m from the gun
3. 120 m from the gun
4. 160 m from the gun

Answer: 3

CORE PRINCIPLE Explosion forces are internal. So the CM still lands 80 m away.

MATH The first piece falls from the top, so it lands at 40 m. Both pieces land together, as both start with zero vertical speed from the same height. For the CM to be at 80 m: $\frac{40+x}{2} = 80$, so $x = 120$ m.

TRAP Option 2 puts both pieces where the CM lands. Option 4 doubles the full range instead of only the second half.

04 The Vector Product

IN NEET AND JEE

- The vector (cross) product of $\vec{a}$ and $\vec{b}$ is a vector $\vec{c}$:

$$\vec{c} = \vec{a} \times \vec{b}, \quad c = ab\sin\theta$$

- θ is the angle between $\vec{a}$ and $\vec{b}$.
- $\vec{c}$ is perpendicular to the plane of $\vec{a}$ and $\vec{b}$.
- Its direction follows the **right-hand rule**. Curl your fingers from $\vec{a}$ to $\vec{b}$. Your thumb points along $\vec{c}$.
- The magnitude $ab\sin\theta$ equals the area of the parallelogram made by $\vec{a}$ and $\vec{b}$.

Properties

- **Not commutative:** $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$. Order matters.
- **Distributive:** $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$.
- $\vec{a} \times \vec{a} = 0$, because $\sin 0 = 0$.
- Unit vectors: $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$. Reverse the order and the sign flips.
- $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$.

In components

- Write it as a determinant:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$\vec{a} \times \vec{b} = (a_y b_z - a_z b_y)\hat{i} + (a_z b_x - a_x b_z)\hat{j} + (a_x b_y - a_y b_x)\hat{k}$$

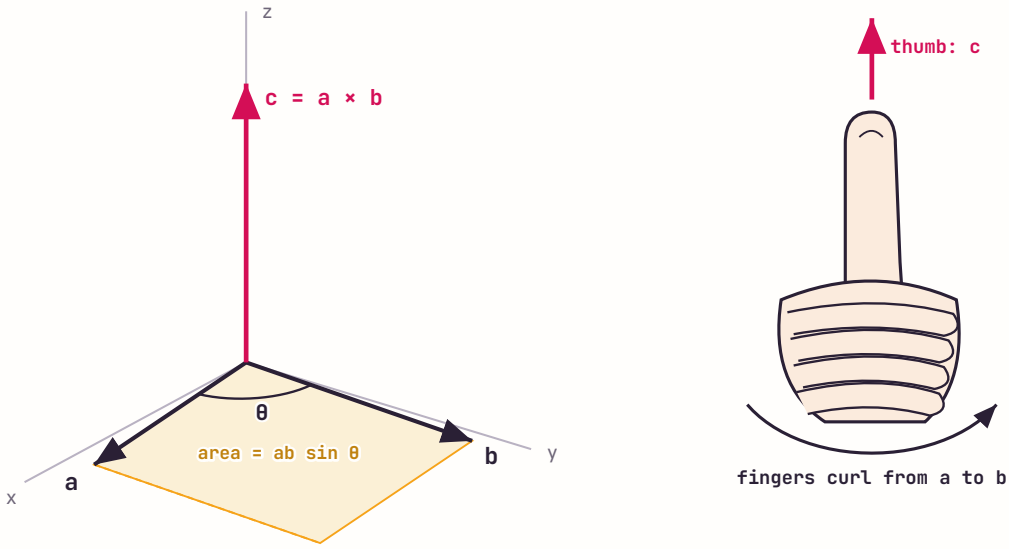


Fig 5 · Cross product. The size of a × b is the parallelogram's area; its direction is set by the right hand.



TRAP ALERT

TRAP ALERT

- The dot product gives a number, using $\cos\theta$. The cross product gives a vector, using $\sin\theta$. Mixing them up is the most common slip in torque questions.

READ NEXT **Vectors notes: dot product and components** →

05 Angular Velocity and Angular Acceleration

IN NEET AND JEE

- In rotation about a fixed axis, every particle moves in a circle.
- All particles turn through the same angle in the same time.
- The rate of change of this angle is the **angular velocity**, $\omega = \frac{d\theta}{dt}$.
- A particle at perpendicular distance r from the axis has linear speed:

$$v = \omega r$$

- So particles farther from the axis move faster. Particles on the axis do not move.
- Angular velocity is a vector. It points **along the axis**, given by the right-hand rule.
- In vector form: $\vec{v} = \vec{\omega} \times \vec{r}$.
- For a fixed axis, $\vec{\omega}$ keeps the same direction. Only its size can change.
- **Angular acceleration** is $\alpha = \frac{d\omega}{dt}$. For a fixed axis it also points along the axis.

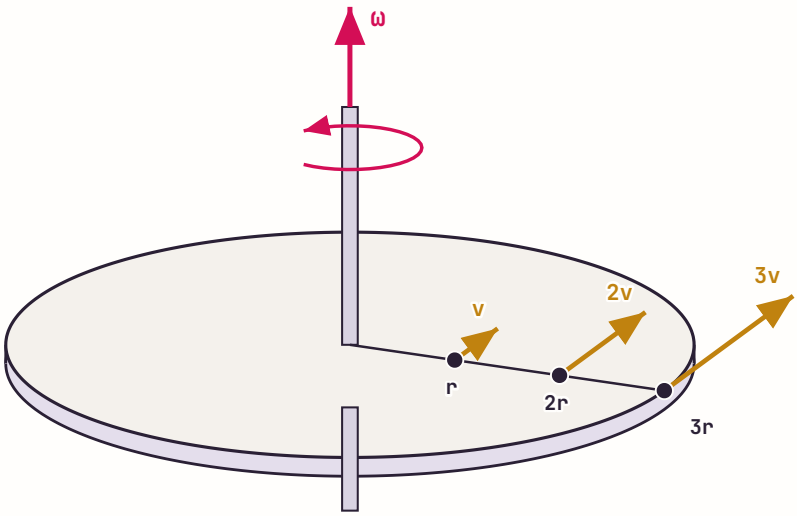


Fig 7 · Angular velocity. $v = \omega r$: twice as far, twice as fast.



THINK IT THROUGH

SANITY CHECK

- A point on a rotating disc that is speeding up has two accelerations. Tangential: αr. Towards the centre: $\omega^2 r$. They are perpendicular, so add them as vectors.

READ NEXT **Circular Motion notes: centripetal and tangential acceleration** →

06 Torque and Angular Momentum

IN NEET AND JEE · REGULAR · *torque and angular acceleration, 14% of JEE Main mechanics questions*

Torque (moment of force)

- A force can turn a body. Its turning effect is the **torque**:

$$\vec{\tau} = \vec{r} \times \vec{F}, \quad \tau = rF\sin\theta$$

- $\vec{r}$ runs from the axis (or origin) to the point where the force acts.
- Two useful ways to read $rF\sin\theta$:
  - $r_\perp F$, where $r_\perp = r\sin\theta$ is the **lever arm**: the perpendicular distance from the axis to the force's line.
  - $rF_\perp$, where $F_\perp = F\sin\theta$ is the part of the force perpendicular to $\vec{r}$.

- Torque is zero if the force passes through the axis, or if $\vec{r} = 0$.
- SI unit: N m. Dimensions: $[ML^2T^{-2}]$.



TRAP ALERT

- Torque and work have the same dimensions, $[ML^2T^{-2}]$. But torque is a vector and work is a scalar. Do not write torque in joules.

Angular momentum of a particle

- For a particle with momentum $\vec{p}$ at position $\vec{r}$:

$$\vec{l} = \vec{r} \times \vec{p}, \quad l = rp\sin\theta = r_{\perp} p$$

- A particle moving in a **straight line** can have angular momentum about a point. It only needs a lever arm.
- The link with torque:

$$\frac{d\vec{l}}{dt} = \vec{\tau}$$

- This is the rotational twin of $\frac{d\vec{p}}{dt} = \vec{F}$.

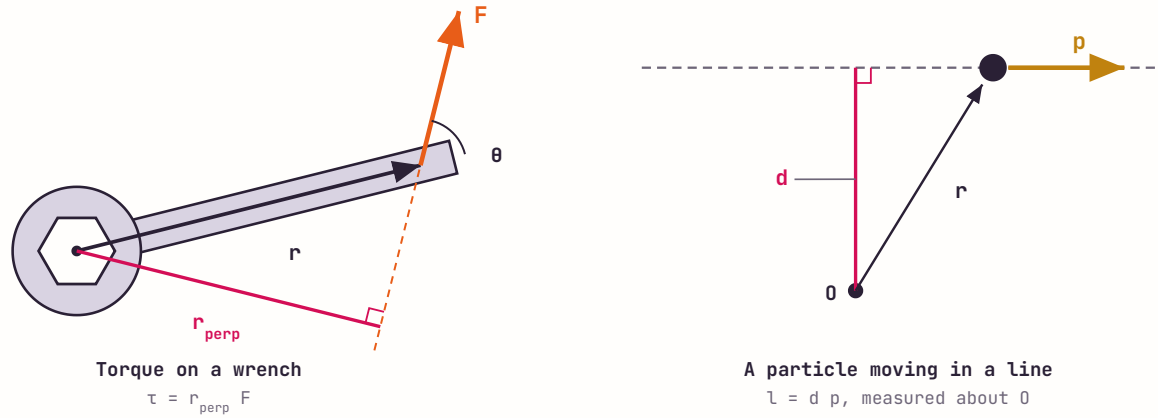


Fig 6 · Torque and angular momentum. Only the perpendicular distance to the line of action counts.



TRAP ALERT

TRAP ALERT

- For a point particle, use $L = r_\perp p$, not $I\omega$. This slip appears in JEE Main papers.

A system of particles

- Total angular momentum $\vec{L} = \sum \vec{l}_i$.
- Internal torques cancel in pairs, when internal forces act along the lines joining the particles.
- So only external torque changes $\vec{L}$:

$$\frac{d\vec{L}}{dt} = \vec{\tau}_{ext}$$

- **Conservation of angular momentum:** if $\vec{\tau}_{ext} = 0$, then $\vec{L}$ is constant.

Q4 Worked question

JEE MAIN LEVEL 2

A force $\vec{F} = (2\hat{i} - 3\hat{j} + \hat{k})$ N acts at the point $\vec{r} = (\hat{i} + 2\hat{j} - \hat{k})$ m. What is the magnitude of the torque about the origin?

1. $\sqrt{59}$ N m
2. $\sqrt{84}$ N m
3. 5 N m
4. 7 N m

Answer: 1

CORE PRINCIPLE $\vec{\tau} = \vec{r} \times \vec{F}$, then take the magnitude.

MATH $\tau_x = (2)(1) - (-1)(-3) = -1$. $\tau_y = (-1)(2) - (1)(1) = -3$. $\tau_z = (1)(-3) - (2)(2) = -7$. So $|\vec{\tau}| = \sqrt{1 + 9 + 49} = \sqrt{59}$ N m.

TRAP Option 2 multiplies $|\vec{r}||\vec{F}|$ and forgets $\sin\theta$. Option 3 is the size of the dot product. Option 4 keeps only the z-component.

07 Equilibrium of a Rigid Body

IN NEET AND JEE

Two conditions

- A rigid body is in **mechanical equilibrium** when both its momenta stay constant.

  1. **Translational equilibrium:** total external force is zero, $\sum \vec{F} = 0$.
  2. **Rotational equilibrium:** total external torque is zero, $\sum \vec{\tau} = 0$.

- One condition can hold without the other.

Couple

- A **couple** is a pair of equal and opposite forces with different lines of action.
- The net force is zero, so the CM does not accelerate.
- But the net torque is not zero. So the body turns.
- Examples: turning a steering wheel; turning a tap; a compass needle in a uniform magnetic field.

Principle of moments (the lever)

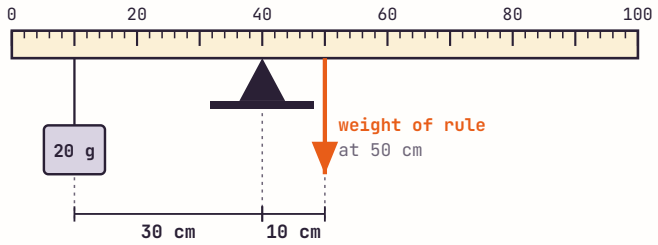
- A lever is a light rod pivoted at a point called the **fulcrum**.
- At balance, the torques about the fulcrum cancel:

$$F_1 d_1 = F_2 d_2$$

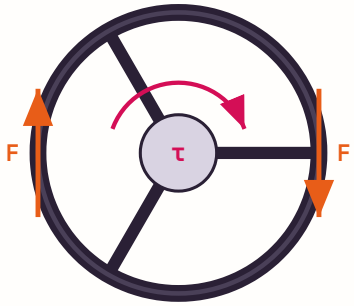
- F_1 is the load and d_1 its arm. F_2 is the effort and d_2 its arm.
- **Mechanical advantage** $= \frac{F_1}{F_2} = \frac{d_2}{d_1}$. A longer effort arm lifts a bigger load.

Centre of gravity

- The **centre of gravity (CG)** is the point where the total gravitational torque on the body is zero.
- If g is the same everywhere over the body, the CG and the CM are the same point.
- For a very tall body, g changes with height. Then CG and CM differ slightly.
- A body hung from its CG balances in any position.



Balance: 20 × 30 = m × 10



A couple
net force 0, net torque not 0

Fig 8 · Equilibrium. Balance needs zero net torque; a couple turns without pushing.

Q5 Worked question

NEET LEVEL 2

A uniform metre rule balances on a knife edge at the 40 cm mark. A 20 g mass hangs at the 10 cm mark. What is the mass of the rule?

1. 60 g
2. 20 g
3. 12 g
4. 80 g

Answer: 1

CORE PRINCIPLE A uniform rule's weight acts at its centre, the 50 cm mark. Take moments about the knife edge.

MATH $20 \times (40 - 10) = m \times (50 - 40)$, so $600 = 10m$, and $m = 60$ g.

TRAP Option 2 assumes equal masses balance at any point. Option 3 measures the rule's arm from the 0 cm end (50 cm). Option 4 measures the hanging mass's arm from the rule's centre (40 cm).

08 Moment of Inertia

IN NEET AND JEE TOP TOPIC *22% of JEE Main mechanics questions*

Where it comes from

- A body rotates about a fixed axis with angular speed ω.
- A particle at distance r_i has speed $v_i = r_i\omega$.

- Total kinetic energy:

$$K = \sum \tfrac{1}{2} m_i r_i^2 \omega^2 = \tfrac{1}{2} I \omega^2$$

- Here $I = \sum m_i r_i^2$ is the **moment of inertia** about that axis.
- It plays the role of mass in rotation. It measures how hard it is to change the body's rotation.

What it depends on

- The mass of the body.
- How that mass is spread around the axis. Mass farther from the axis counts much more, because of r^2.
- The **position and direction of the axis**. The same body has different I about different axes.
- It does **not** depend on how fast the body spins.

Radius of gyration

- Write $I = Mk^2$. Then k is the **radius of gyration**.
- It is the distance at which all the mass, packed into one point, would give the same I.
- k also depends on the axis.

Standard values (state them; derivations are not needed)

BODY	AXIS	I
Thin ring, radius R	Through centre, perpendicular to plane	MR^2
Thin ring	Any diameter	$\frac{1}{2}MR^2$
Thin rod, length L	Through centre, perpendicular to rod	$\frac{1}{12}ML^2$
Thin rod	Through one end, perpendicular to rod	$\frac{1}{3}ML^2$
Disc, radius R	Through centre, perpendicular to plane	$\frac{1}{2}MR^2$
Disc	Any diameter	$\frac{1}{4}MR^2$
Hollow cylinder	Its own axis	MR^2
Solid cylinder	Its own axis	$\frac{1}{2}MR^2$
Solid sphere	Any diameter	$\frac{2}{5}MR^2$
Hollow sphere (thin shell)	Any diameter	$\frac{2}{3}MR^2$

- The last row is not in the NCERT table. JEE uses it often, so learn it.

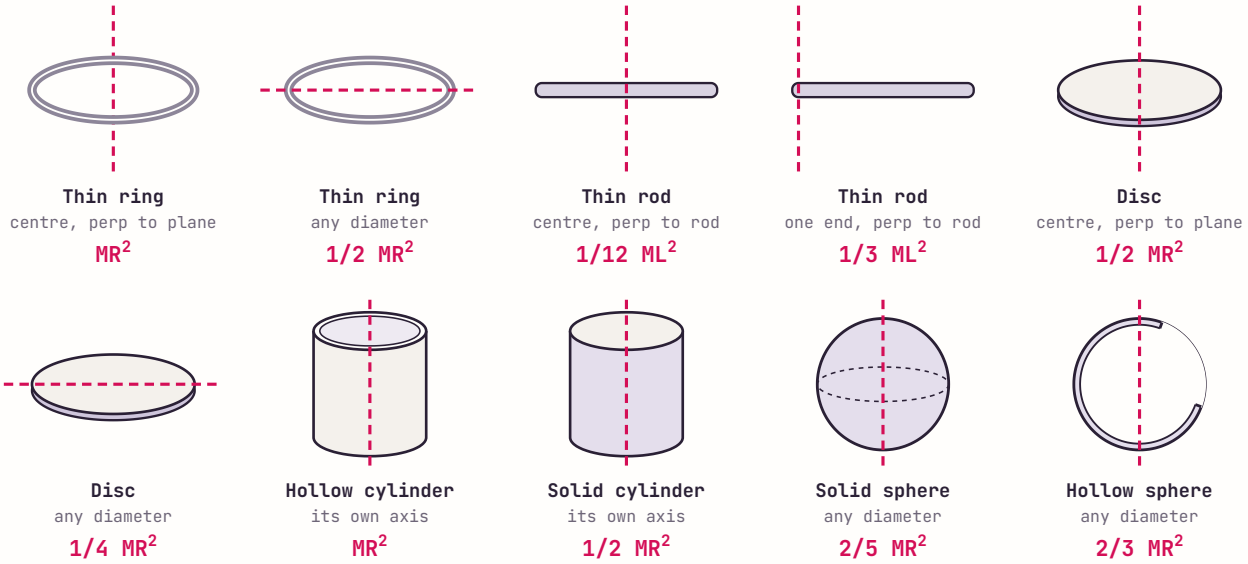


Fig 9 · Moment of inertia gallery. Same body, different axis, different I.



THINK IT THROUGH

SANITY CHECK

- A ring and a disc of the same mass and radius: the ring has twice the I. All the ring's mass sits at the rim. The disc spreads some mass close to the axis.

09 Theorems of Parallel and Perpendicular Axes

IN NEET AND JEE · NEARLY EVERY SHIFT · *JEE Main, parallel axis theorem*

NOTE

NCERT removed this section in the rationalised book. But both the NEET 2026 and JEE Main 2026 syllabi still list "parallel and perpendicular axes theorems and their applications". So learn it fully.

Perpendicular axis theorem (planar bodies only)

- Take a flat body (lamina) in the x-y plane.
- The z-axis is perpendicular to the lamina. All three axes meet at one point.

$$I_z = I_x + I_y$$

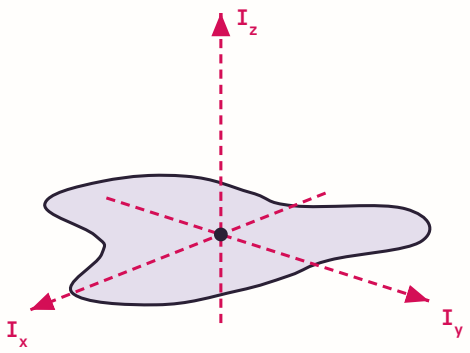
- **Example:** a ring has $I_z = MR^2$. By symmetry, $I_x = I_y$. So each diameter has $I = \frac{1}{2}MR^2$.
- Similarly, a disc's diameter has $I = \frac{1}{4}MR^2$.

Parallel axis theorem (any body)

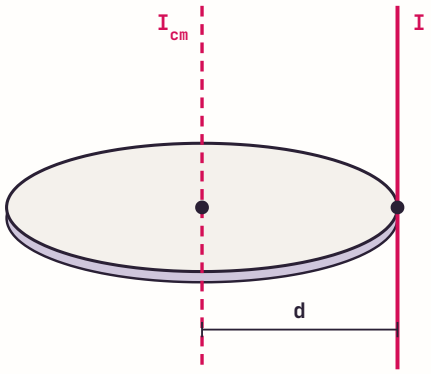
- Take an axis through the CM, with moment of inertia I_{cm}.
- Take any parallel axis at distance d:

$$I = I_{cm} + Md^2$$

- **Example:** a rod about one end: $\frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{3}ML^2$.
- **Example:** a disc about a tangent perpendicular to its plane: $\frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$.



$I_z = I_x + I_y$, flat bodies only



$I = I_{cm} + Md^2$

Fig 10 · Axes theorems. Perpendicular axes for flat bodies; parallel axes from the CM.


TRAP ALERT

TRAP ALERT

- The parallel axis theorem must start from an axis through the **CM**. You cannot jump between two non-CM axes directly. Go to the CM axis first, then out again.


TRAP ALERT

TRAP ALERT

- The perpendicular axis theorem works only for **flat** bodies. It fails for a sphere or a cylinder.

Q6 Worked question

JEE MAIN LEVEL 2

A uniform rod has mass M and length L. It rotates about a perpendicular axis through a point $L/4$ from one end. What is its moment of inertia?

1. $\frac{1}{48}ML^2$
2. $\frac{7}{48}ML^2$
3. $\frac{1}{16}ML^2$
4. $\frac{1}{3}ML^2$

Answer: 2

CORE PRINCIPLE The axis is $L/4$ from the centre. Use the parallel axis theorem from the CM axis.

MATH $I = \frac{1}{12}ML^2 + M\left(\frac{L}{4}\right)^2 = \frac{4+3}{48}ML^2 = \frac{7}{48}ML^2$.

TRAP Option 1 subtracts Md^2 instead of adding. Option 3 keeps only Md^2. Option 4 is the end-axis value, used without checking where the axis is.

Q7 Worked question

NEET JEE MAIN LEVEL 2

A thin ring has moment of inertia MR^2 about the axis through its centre, perpendicular to its plane. What is its moment of inertia about a tangent that lies **in its plane**?

1. $\frac{1}{2}MR^2$
2. $2MR^2$
3. $\frac{3}{2}MR^2$
4. $\frac{1}{4}MR^2$

Answer: 3

CORE PRINCIPLE Two steps. Perpendicular axis theorem gives the diameter. Parallel axis theorem moves to the tangent.

MATH $I_{diameter} = \frac{1}{2}MR^2$. The in-plane tangent is parallel to a diameter, at distance R. So $I = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$.

TRAP Option 1 stops after the first step. Option 2 uses the tangent **perpendicular** to the plane. Option 4 uses the disc's diameter value.

10 Kinematics of Rotation about a Fixed Axis

IN NEET AND JEE

- With constant angular acceleration α, the equations match those of linear motion:

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

- θ is the angle turned, in **radians**.
- One revolution $= 2\pi$ rad. To convert rpm to rad/s, multiply by $\frac{2\pi}{60}$.

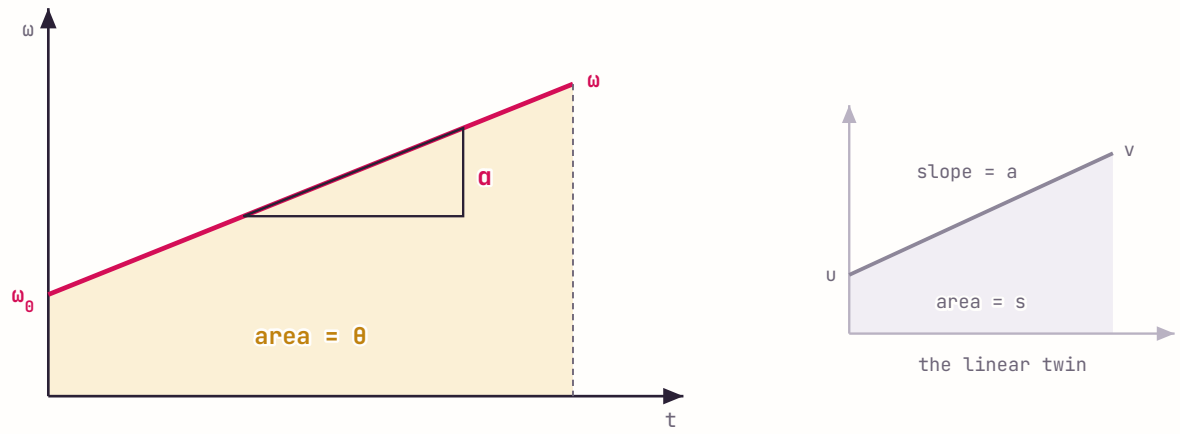


Fig 11 · Rotational kinematics graph. Slope gives α; area gives θ, just like v-t.



TRAP ALERT

TRAP ALERT

- Convert revolutions to radians and rpm to rad/s before using these equations. Our JEE Main analysis lists rpm conversion as a repeated trap.

11 Dynamics of Rotation about a Fixed Axis

IN NEET AND JEE REGULAR

Work, power and the second law

- A torque τ turning a body through a small angle $d\theta$ does work $dW = \tau\, d\theta$.
- Power: $P = \tau\omega$.
- Work done by the torque goes into rotational kinetic energy.
- This gives the rotational form of Newton's second law:

$$\tau = I\alpha$$

- Only the torque component along the fixed axis matters. Other components are cancelled by the bearings.

The twin table

LINEAR MOTION	ROTATION ABOUT A FIXED AXIS
Displacement x	Angular displacement θ
Velocity v	Angular velocity ω
Acceleration a	Angular acceleration α
Mass M	Moment of inertia I
Force $F = Ma$	Torque $\tau = I\alpha$
Work $dW = F\, ds$	Work $dW = \tau\, d\theta$
Kinetic energy $\frac{1}{2}Mv^2$	Kinetic energy $\frac{1}{2}I\omega^2$
Power $P = Fv$	Power $P = \tau\omega$
Momentum $p = Mv$	Angular momentum $L = I\omega$

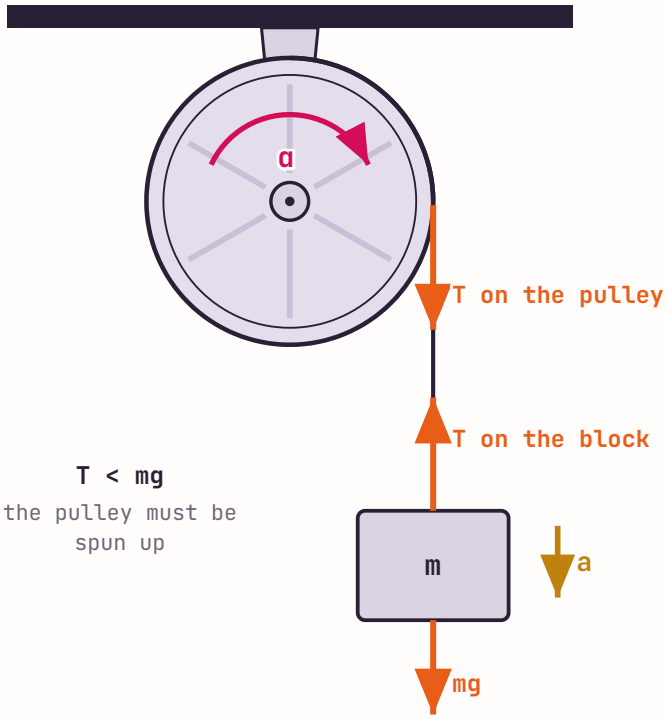


Fig 12 · Pulley with mass. The pulley's inertia makes the tension less than the weight.



TRAP ALERT

TRAP ALERT

- A pulley with mass needs torque to speed up. So the string tension is less than the hanging weight. A massless-pulley answer is wrong here.

Q8 Worked question

JEE MAIN LEVEL 2

A light string is wound on a uniform disc pulley of mass 2 kg and radius 0.2 m. The pulley turns freely on a fixed horizontal axle. A 1 kg block hangs from the string. Find the tension in the string. Take $g = 10\ \text{m/s}^2$.

1. 0 N
2. 10 N
3. 5 N
4. 6.7 N

Answer: 3

CORE PRINCIPLE Block: $mg - T = ma$. Pulley: $TR = I\alpha$ with $I = \frac{1}{2}MR^2$ and $a = \alpha R$.

MATH From the pulley, $T = \frac{1}{2}Ma = a$ (for $M = 2$ kg). From the block, $10 - a = a$. So $a = 5\ \text{m/s}^2$ and $T = 5$ N.

TRAP Option 4 treats the pulley as a ring, $I = MR^2$. Option 1 treats it as massless. Option 2 assumes the block is at rest.

Q9 Worked question

NEET LEVEL 2

A flywheel with $I = 0.2$ kg m^2 spins at 30 rad/s. A constant braking torque stops it in exactly 15 revolutions. What is the torque?

1. 6.0 N m
2. 1.9 N m
3. 3.0 N m
4. 0.95 N m

Answer: 4

CORE PRINCIPLE Work done by the torque removes all the rotational kinetic energy. So $\tau\theta = \frac{1}{2}I\omega^2$.

MATH $K = \frac{1}{2}(0.2)(30)^2 = 90$ J. $\theta = 15 \times 2\pi = 30\pi$ rad. $\tau = \frac{90}{30\pi} \approx 0.95$ N m.

TRAP Option 1 uses 15 rad, not 15 revolutions. Option 3 uses 30 instead of 30π. Option 2 forgets the $\frac{1}{2}$ in kinetic energy.

READ NEXT **Work, Power and Energy notes: the linear twin of these results** →

12 Angular Momentum about a Fixed Axis, and Its Conservation

IN NEET AND JEE REGULAR *8% of JEE Main mechanics questions*

- For rotation about a fixed axis, the angular momentum along the axis is $L = I\omega$.
- Then $\frac{dL}{dt} = \tau$. If I is constant, this is $\tau = I\alpha$ again.
- **If the external torque is zero, $L = I\omega$ stays constant.**
- If I changes, ω must change the other way.

Everyday examples

- A person on a swivel chair spins with arms stretched out. When she pulls her arms in, I falls and she spins faster.
- A skater or a ballet dancer pulls in her arms to spin faster.
- A diver curls up in mid-air to turn more somersaults. Then stretches out before entering the water to slow the turning.

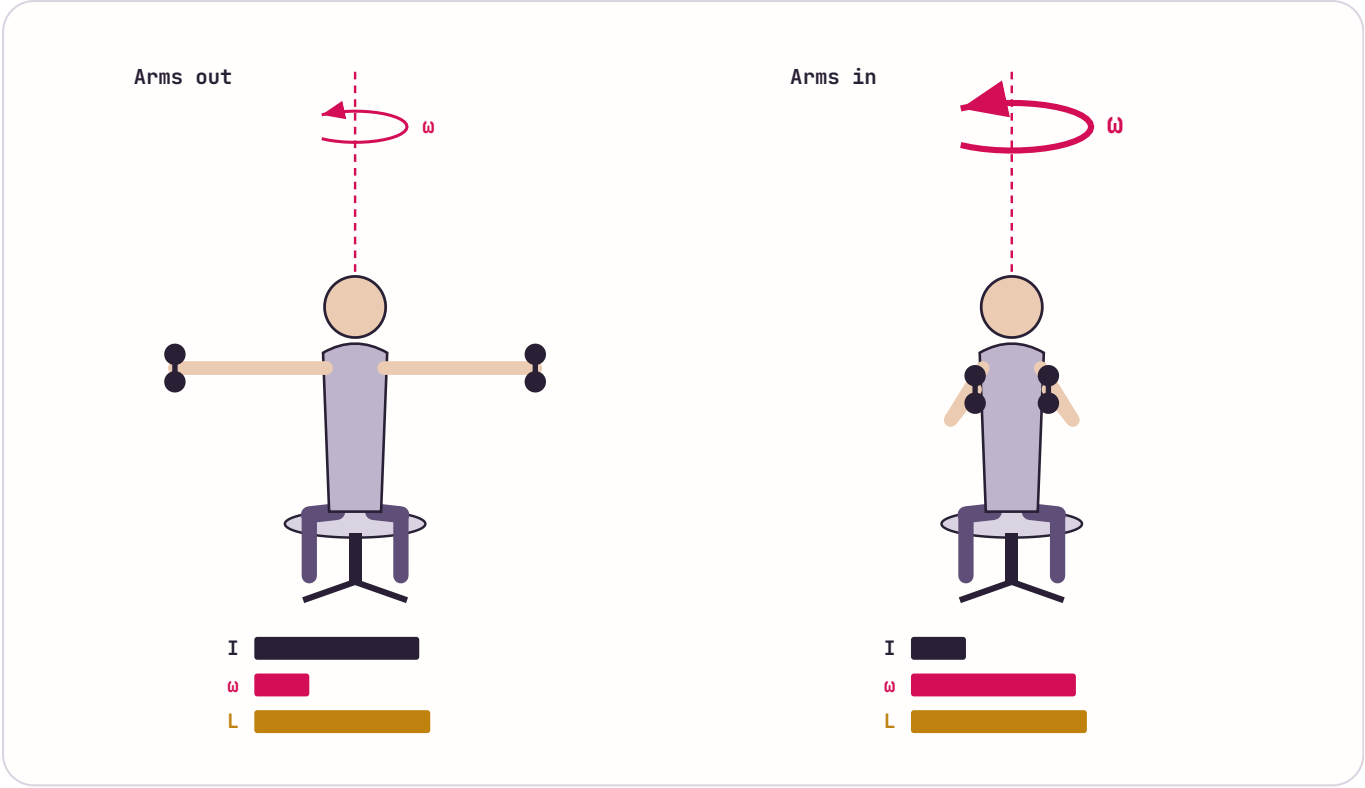


Fig 13 · Conservation of angular momentum. I falls, ω rises, L stays the same.



WAIT, REALLY?

MYTH

If angular momentum is conserved, rotational kinetic energy is conserved too.

SCIENCE

$K = \frac{L^2}{2I}$. When I falls with L fixed, K **rises**. The extra energy comes from the work her muscles do pulling the arms in.

Q10 Worked question

NEET LEVEL 2

A student stands on a turntable spinning freely. With arms out, the system's moment of inertia is 6 kg m^2. He pulls his arms in, and it becomes 2 kg m^2. What happens to the rotational kinetic energy?

1. It stays the same
2. It becomes 3 times
3. It becomes 9 times
4. It becomes one third

Answer: 2

CORE PRINCIPLE No external torque, so L is constant. Write $K = \frac{L^2}{2I}$.

MATH I falls to one third. So ω triples and $K = \frac{L^2}{2I}$ triples.

TRAP Option 1 assumes energy is conserved alongside L. Option 3 squares the speed change but forgets that I fell. Option 4 inverts the ratio.

Q11 Worked question

JEE MAIN NUMERICAL LEVEL 2

Disc A ($I = 0.4$ kg m^2) spins at 50 rad/s about a frictionless axle. Disc B ($I = 0.1$ kg m^2) is at rest on the same axle. B is dropped onto A, and they spin together. How much kinetic energy is lost, in joules? Enter an integer.

Answer: 100

CORE PRINCIPLE The friction between the discs is internal. So angular momentum is conserved, but kinetic energy is not.

MATH $\omega_f = \frac{0.4 \times 50}{0.4 + 0.1} = 40$ rad/s. $K_i = \frac{1}{2}(0.4)(50)^2 = 500$ J. $K_f = \frac{1}{2}(0.5)(40)^2 = 400$ J. Loss $= 100$ J.

TRAP Students who conserve kinetic energy find a different final speed and zero loss. This is a rotational "perfectly inelastic collision".

13 Rolling Motion

JEE MAIN AND ADVANCED REGULAR *12% of JEE Main mechanics questions*

NOTE

Rolling was removed from the rationalised NCERT book and is not in the NEET 2026 syllabus. NEET students can skip this section. JEE students cannot.

Rolling without slipping

- A rolling wheel translates and rotates at the same time.
- Its CM moves forward with speed v_{cm}.
- At the point of contact, the ground does not slide under the wheel. So the contact point is **momentarily at rest**.
- This gives the **rolling condition**:

$$v_{cm} = R\omega$$

- Velocity of any point = velocity of CM + velocity due to rotation about the CM.
  - Contact point: $v_{cm} - R\omega = 0$.
  - Centre: v_{cm}.
  - Top point: $v_{cm} + R\omega = 2v_{cm}$.

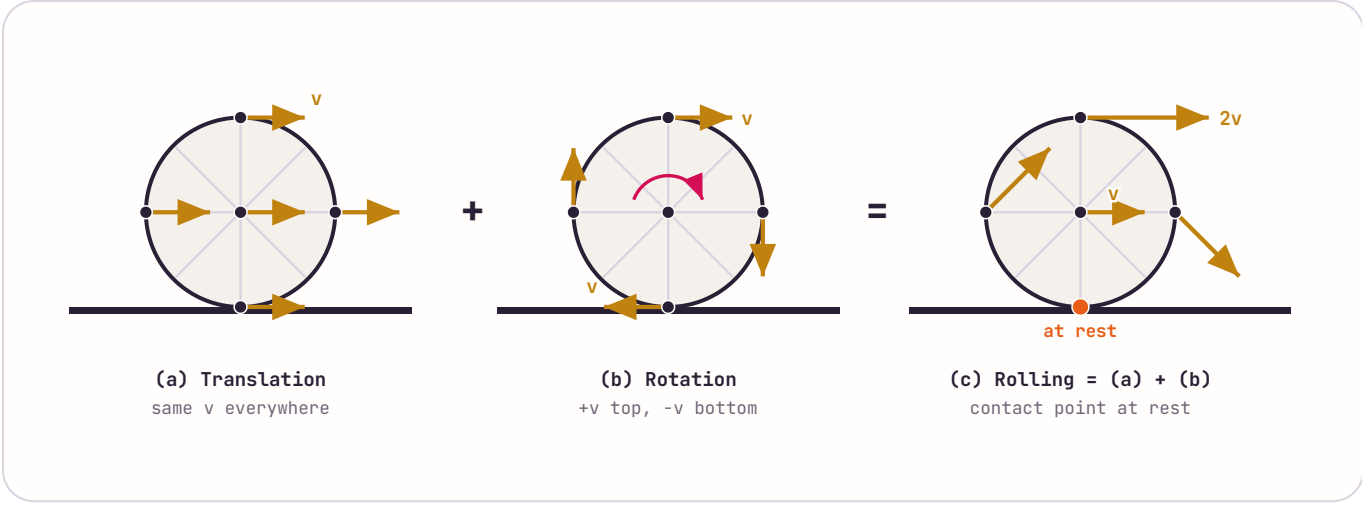
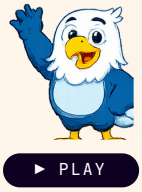


Fig 14 · Rolling. Add translation and rotation: the top moves at 2v, the bottom is still.



HAVING DIFFICULTY SEEING WHY THE TOP OF A WHEEL MOVES FASTEST?

Play it.

This chapter is a set of short games in the Logic Bloom app.
logicbloom.co.in/download · Free to start.

SCAN

Kinetic energy of rolling

$$K = \tfrac{1}{2}Mv_{cm}^2 + \tfrac{1}{2}I_{cm}\omega^2 = \tfrac{1}{2}Mv_{cm}^2\left(1 + \frac{k^2}{R^2}\right)$$

- k is the radius of gyration about the CM axis.
- $\frac{k^2}{R^2}$ is 1 for a ring, $\frac{1}{2}$ for a disc or solid cylinder, $\frac{2}{5}$ for a solid sphere.

Rolling down an incline

$$a = \frac{g\sin\theta}{1 + k^2/R^2}, \qquad v = \sqrt{\frac{2gh}{1 + k^2/R^2}}$$

- A smaller $\frac{k^2}{R^2}$ means a bigger share of energy goes into translation.
- So a solid sphere reaches the bottom first, then a disc, then a ring. Mass and radius do not matter.
- Friction is needed for rolling, but static friction at the contact point does **no work**. So mechanical energy is conserved.



TRAP ALERT

TRAP ALERT

- Rolling up a slope with the same starting speed, bodies with a larger $\frac{k^2}{R^2}$ climb **higher**. The heights for sphere, disc and ring are in the ratio $14 : 15 : 20$. This exact ratio appears in JEE Main papers.

Q12 Worked question

JEE MAIN LEVEL 2

A solid sphere, a disc and a ring all roll without slipping. Each starts up the same incline with the same speed v. Find the ratio of the heights they reach (sphere : disc : ring).

1. $1 : 1 : 1$
2. $20 : 15 : 14$
3. $4 : 5 : 10$
4. $14 : 15 : 20$

Answer: 4

CORE PRINCIPLE All the kinetic energy turns into potential energy. So $Mgh = \frac{1}{2}Mv^2\left(1 + \frac{k^2}{R^2}\right)$, and $h \propto 1 + \frac{k^2}{R^2}$.

MATH Sphere 1.4, disc 1.5, ring 2. Multiply by 10: $14 : 15 : 20$.

TRAP Option 1 ignores rotational energy. Option 2 inverts the ratio. Option 3 uses $\frac{k^2}{R^2}$ alone and forgets the translational 1.

READ NEXT **Friction notes: static friction and when it does no work** →

14 JEE Advanced Challenge Set

JEE ADVANCED MAINLY

Five problems. Each needs calculus, a second chapter, a surprising result, an optimisation, or a solution path you build yourself. Take 6 to 8 minutes each.

Q13 Worked question

JEE ADVANCED · NUMERICAL · LEVEL 3

A disc of radius R has surface mass density $\sigma = kr$. Here r is the distance from its centre, and k is a constant. It rolls without slipping down a fixed incline of angle 30°. The axis is through its centre, perpendicular to its face. Friction is enough to prevent slipping. Take $g = 9.8$ m/s^2. Find the acceleration of its centre, in m/s^2, correct to two decimal places.

Answer: 3.06

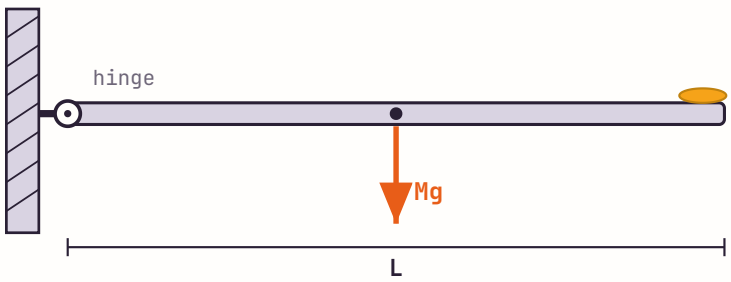
CORE PRINCIPLE The mass is not uniform, so find I by integration. Then use the rolling result.

MATH Use rings of radius r and width dr. $M = \int_0^R kr \cdot 2\pi r\, dr = \frac{2\pi k R^3}{3}$. $I = \int_0^R kr \cdot 2\pi r \cdot r^2\, dr = \frac{2\pi k R^5}{5}$. So $I = \frac{3}{5}MR^2$. Then $a = \frac{g\sin 30^\circ}{1 + 3/5} = \frac{4.9}{1.6} = 3.0625 \approx 3.06$ m/s^2.

TRAP Using a uniform disc gives 3.27. Treating it as a ring gives 2.45. The mass leans towards the rim, so the answer lies between them.

Q14 Worked question

JEE ADVANCED · ONE OR MORE CORRECT · LEVEL 3



A uniform rod hinged at one end, held horizontal.

A uniform rod of mass M and length L is hinged at one end. It is held horizontal and then released. At the instant of release, which of the following is/are correct? (One or more of the following may be correct.)

(A) The angular acceleration of the rod is $\frac{3g}{2L}$.

(B) The free end accelerates downward at $\frac{3g}{2}$, faster than free fall.

(C) The force from the hinge on the rod is $\frac{Mg}{2}$, upward.

(D) Every point of the rod accelerates faster than g.

Answer: (A) and (B)

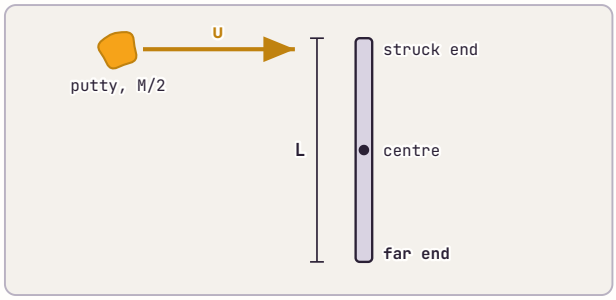
CORE PRINCIPLE Take torques about the hinge. Then use the CM equation for the hinge force.

MATH $\tau = Mg\frac{L}{2}$ and $I = \frac{1}{3}ML^2$, so $\alpha = \frac{3g}{2L}$. Free end: $a = \alpha L = \frac{3g}{2}$. CM: $a_{cm} = \alpha\frac{L}{2} = \frac{3g}{4}$. So $Mg - N = M\frac{3g}{4}$ and $N = \frac{Mg}{4}$.

TRAP (C) guesses that the hinge carries half the weight. (D) is false: a point at distance x has $a = \frac{3gx}{2L}$, which beats g only beyond $x = \frac{2L}{3}$.

Q15 Worked question

JEE ADVANCED LEVEL 3



Top view: a rod on a frictionless table, struck at one end.

A uniform rod of mass M and length L lies at rest on a frictionless horizontal table. It is free, with no pivot. A lump of putty of mass $M/2$ moves on the table at speed u, perpendicular to the rod. It hits one end and sticks. Just after the impact, what is the angular velocity, and which point of the rod is momentarily at rest?

1. $\frac{u}{L}$; the centre of the rod
2. $\frac{u}{L}$; the point $\frac{L}{3}$ from the far (unstruck) end
3. $\frac{6u}{5L}$; the point $\frac{L}{3}$ from the far end
4. $\frac{6u}{5L}$; the far end

Answer: 2

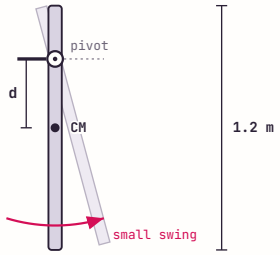
CORE PRINCIPLE With no pivot, the body rotates about its own CM. Conserve linear momentum, and angular momentum about the combined CM.

MATH Combined CM: $d = \frac{(M/2)(L/2)}{3M/2} = \frac{L}{6}$ from the rod's centre, towards the struck end. $v_{cm} = \frac{(M/2)u}{3M/2} = \frac{u}{3}$. Angular momentum about it: $\frac{M}{2}u\left(\frac{L}{2} - \frac{L}{6}\right) = \frac{MuL}{6}$. $I_{cm} = \frac{ML^2}{12} + M\left(\frac{L}{6}\right)^2 + \frac{M}{2}\left(\frac{L}{3}\right)^2 = \frac{ML^2}{6}$. So $\omega = \frac{u}{L}$. The point at rest is $\frac{v_{cm}}{\omega} = \frac{L}{3}$ from the CM, on the far side. That is $\frac{L}{3}$ from the far end.

TRAP Options 3 and 4 take moments about the rod's centre, as if it were pivoted there. That gives $\frac{6u}{5L}$. Option 1 assumes the centre stays still.

Q16 Worked question

JEE ADVANCED LEVEL 3



A rod pivoted at a variable point, swinging.

A uniform rod 1.2 m long can be pivoted at any point along its length. It swings in a vertical plane with small oscillations. Take $g = 10$ m/s^2. For which distance d of the pivot from the centre is the period smallest, and what is that period?

1. $d \approx 0.35$ m; $T \approx 1.65$ s
2. $d \approx 0.60$ m; $T \approx 1.78$ s
3. $d \approx 0.35$ m; $T \approx 1.17$ s
4. $d \approx 0.20$ m; $T \approx 1.78$ s

Answer: 1

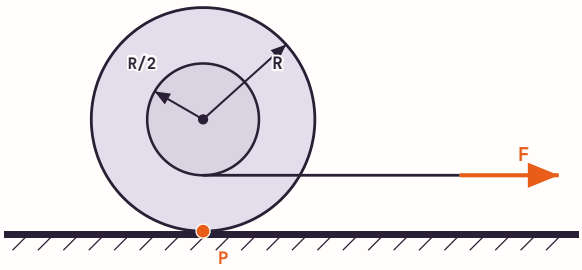
CORE PRINCIPLE This joins rotation with oscillations. A physical pendulum has $T = 2\pi\sqrt{\frac{I}{Mgd}}$, where I is about the pivot.

MATH $I = M(k^2 + d^2)$ with $k^2 = \frac{L^2}{12} = 0.12$ m^2. So $T = 2\pi\sqrt{\frac{k^2 + d^2}{gd}}$. Minimise $\frac{k^2}{d} + d$: it is smallest at $d = k = \sqrt{0.12} \approx 0.35$ m. Then $T = 2\pi\sqrt{\frac{2k}{g}} \approx 1.65$ s.

TRAP Option 2 assumes the end pivot is best. Option 4 is a surprise. Pivots at 0.20 m and 0.60 m give the **same** period. That is because d and $\frac{k^2}{d}$ form a pair. Option 3 drops the factor 2 inside the root.

Q17 Worked question

JEE ADVANCED | ONE OR MORE CORRECT | LEVEL 3



A spool on a rough floor, thread pulled from below the axle.

A spool of mass M has outer radius R and an inner axle of radius $R/2$. Take its moment of inertia about its axis as $\frac{1}{2}MR^2$. It stands on a rough horizontal floor. A thread is wound on the axle. It leaves from the **bottom** of the axle and is pulled horizontally with force F. The spool rolls without slipping. Which of the following is/are correct? (One or more of the following may be correct.)

(A) The spool's centre accelerates towards the pull at $\frac{F}{3M}$.

(B) Friction on the spool is $\frac{2F}{3}$, opposite to the pull.

(C) If the thread left from the **top** of the axle instead, friction on the spool would be zero.

(D) The spool rolls away from the direction of the pull.

Answer: (A), (B) and (C)

CORE PRINCIPLE Take torques about the contact point, which is momentarily at rest. Then use $F + f = Ma$ for friction.

MATH $I_{contact} = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$. The pull's lever arm about the contact is $R - \frac{R}{2} = \frac{R}{2}$. So $F\frac{R}{2} = \frac{3}{2}MR^2 \cdot \frac{a}{R}$, giving $a = \frac{F}{3M}$. Then $f = Ma - F = -\frac{2F}{3}$. For the top of the axle, the lever arm is $\frac{3R}{2}$, so $a = \frac{F}{M}$ and $f = 0$.

TRAP (D) is the famous wrong guess. The pull's line passes **above** the contact point, so the spool turns towards the pull. Students who guess friction's direction first, instead of solving for it, get (B) wrong.

15 Quick Review: What JEE Main Repeats

Frequency bands come from our JEE Main analysis. NEET data for this chapter is coming.

NEARLY EVERY SHIFT — AXES THEOREMS

When can you use $I = I_{cm} + Md^2$?

For any body, but only from an axis through the CM to a parallel axis.

Why it recurs: Our analysis found it in nearly every shift.

NEARLY EVERY SHIFT — AXES THEOREMS

Which bodies obey $I_z = I_x + I_y$?

Only flat (planar) bodies.

Why it recurs: Applied to a sphere, it gives a wrong answer.

TOP TOPIC — MOMENT OF INERTIA

Ring vs disc, same M and R, central perpendicular axis?

Ring MR^2; disc $\frac{1}{2}MR^2$.

Why it recurs: Every composite-shape question starts here.

TOP TOPIC — MOMENT OF INERTIA

Thin rod: centre axis vs end axis?

$\frac{1}{12}ML^2$ and $\frac{1}{3}ML^2$.

Why it recurs: Rod-based composites are a fixed archetype.

TOP TOPIC — MOMENT OF INERTIA

Solid sphere vs hollow sphere about a diameter?

$\frac{2}{5}MR^2$ and $\frac{2}{3}MR^2$.

Why it recurs: Used in rolling and composite questions.

TOP TOPIC — MOMENT OF INERTIA

A disc loses a hole of half its radius. What fraction of mass goes?

One quarter.

Why it recurs: A named trap in our analysis.

TOP TOPIC — RADIUS OF GYRATION

What does k mean, and does it depend on the axis?

$I = Mk^2$; yes, it depends on the axis.

Why it recurs: Often asked as a ratio between two axes.

REGULAR TORQUE

What is the lever arm?

The perpendicular distance from the axis to the force's line.

Why it recurs: $\tau = r_\perp F$ is the fastest route.

REGULAR TORQUE

How do you change rpm to rad/s?

Multiply by $\frac{2\pi}{60}$.

Why it recurs: A named trap in our analysis.

REGULAR TORQUE

A pulley with mass: is the tension equal to the hanging weight?

No. Use $\tau = I\alpha$ on the pulley.

Why it recurs: Pulley-with-mass questions are frequent.

REGULAR ANGULAR MOMENTUM

A skater pulls her arms in. What happens to ω and to KE?

Both increase. Her muscles supply the energy.

Why it recurs: Students wrongly conserve KE.

REGULAR ANGULAR MOMENTUM

Two discs stick on one axle. What is conserved?

Angular momentum, not kinetic energy.

Why it recurs: A favourite numerical-answer setup.

REGULAR ANGULAR MOMENTUM

Angular momentum of a point particle about a point?

$r_\perp p$, not $I\omega$.

Why it recurs: A named trap in our analysis.

REGULAR ROLLING (JEE)

Speed of the contact point and the top of a rolling wheel?

0 and $2v_{cm}$.

Why it recurs: Velocity-of-a-point questions.

REGULAR ROLLING (JEE)

Heights reached by sphere, disc, ring with the same speed?

14 : 15 : 20.

Why it recurs: A named trap in our analysis.

REGULAR

ROLLING (JEE)

Which rolls down fastest: ring, disc or solid sphere?

> **Solid sphere, then disc, then ring.**

Why it recurs: Smallest k^2/R^2 wins.

OCCASIONAL

CENTRE OF MASS

Where does the CM of an exploding shell go?

> **Along the original parabola.**

Why it recurs: Explosion forces are internal.

OCCASIONAL

CENTRE OF MASS

Where is the CM of two unequal masses?

> **Closer to the heavier one.**

Why it recurs: The basis of every CM question.

OCCASIONAL

EQUILIBRIUM

What does a couple do?

> **Turns the body without moving its CM.**

Why it recurs: Net force zero, net torque not zero.

OCCASIONAL

KINEMATICS

What unit must θ be in for $\omega^2 = \omega_0^2 + 2\alpha\theta$?

> **Radians.**

Why it recurs: Revolutions must be multiplied by 2π.

★ Rotational Motion · Fact Sheet

One page for the night before the exam.

CENTRE OF MASS

$X = \frac{\sum m_i x_i}{M}$ · **continuous:** $X = \frac{1}{M}\int x\,dm$

$M\vec{a}_{cm} = \vec{F}_{ext}$ · $\vec{P} = M\vec{V}_{cm}$

Hole as negative mass; radius $R/2$ removes $M/4$

VECTOR PRODUCT

$|\vec{a} \times \vec{b}| = ab\sin\theta$

Right-hand rule, $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

TORQUE AND ANGULAR MOMENTUM

$\vec{\tau} = \vec{r} \times \vec{F}$, $\tau = r_\perp F$ · $\vec{l} = \vec{r} \times \vec{p}$

$\frac{d\vec{l}}{dt} = \vec{\tau}_{ext}$ · $\tau_{ext} = 0$ means L constant

EQUILIBRIUM

$\sum \vec{F} = 0$ **and** $\sum \vec{\tau} = 0$

Lever: $F_1 d_1 = F_2 d_2$

MOMENT OF INERTIA

$I = Mk^2$ · $I = I_{cm} + Md^2$ · $I_z = I_x + I_y$ **(flat bodies)**

Ring MR^2 · disc $\frac{1}{2}MR^2$ · rod (centre) $\frac{1}{12}ML^2$ · rod (end) $\frac{1}{3}ML^2$

Solid sphere $\frac{2}{5}MR^2$ · hollow sphere $\frac{2}{3}MR^2$ · ring diameter $\frac{1}{2}MR^2$ · disc diameter $\frac{1}{4}MR^2$

ROTATION ABOUT A FIXED AXIS

$\tau = I\alpha$ · $K = \frac{1}{2}I\omega^2$ · $L = I\omega$

$\omega = \omega_0 + \alpha t$ · $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ · $\omega^2 = \omega_0^2 + 2\alpha\theta$

$W = \tau\theta$ · $P = \tau\omega$

ROLLING (JEE)

$v_{cm} = R\omega$ · **contact point at rest · top moves at** $2v_{cm}$

$K = \frac{1}{2}Mv^2(1 + k^2/R^2)$ · $a = \frac{g\sin\theta}{1 + k^2/R^2}$

k^2/R^2: ring 1, disc $\frac{1}{2}$, solid sphere $\frac{2}{5}$

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★ STILL CAN'T PICTURE THE SPIN?

Play it. Then practice it.

Open Rotational Motion in the Logic Bloom app. Play the chapter as short games, then practise the questions exams repeat. Free to start.

SCAN → DO · READ · VERIFY