LOGIC BLOOM · JEE PHYSICS

Relative Motion

Kinematics · **Concept-Clearing Question Set** · Main & Advanced

Ten problems that isolate every relative-motion trap: overtaking, river crossings, rain and umbrella, closest approach, drift optimisation, and projectiles in the relative frame. Each one solved by Core principle, Working, and the Trap that catches most students.

10 Problems | Main + Advanced | Full Solutions | Vector Method | Revision Recap

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0 The Toolkit

Read this first. Every problem in this set collapses to one vector idea.

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

"Velocity of A with respect to B" means: sit on B, treat yourself as still, and watch A. The same subtraction holds for position and acceleration. Five ideas clear almost everything:

1 **Same-line motion.** Same direction subtracts speeds, opposite direction adds them. To overtake or cross, the relative displacement equals the **sum** of the two lengths.

2 **River crossing.** Shortest time means point straight across. Zero drift means point upstream to cancel the current. These are two different headings, not one.

3 **Rain and man.** The rain you feel is $\vec{v}_{rain} - \vec{v}_{man}$. Tilt the umbrella along that vector.

4 **Closest approach.** Freeze one body. The other moves in a straight line at $\vec{v}_{rel}$. Minimum distance is the perpendicular distance from the frozen body to that line.

5 **Two projectiles.** Both share acceleration g, so $\vec{a}_{rel} = 0$. Relative velocity is constant, so one projectile moves in a **straight line** as seen from the other.

1 One-Dimensional Relative Motion

Q1 Overtaking trains

JEE MAIN

Two trains, each 120 m long, run in the **same direction** on parallel tracks at 90 km/h and 72 km/h. How long does the faster train take to completely overtake the slower one?

CORE PRINCIPLE

Overtaking is a relative-displacement problem. Ride on the slow train; the fast train must cover its own length plus the slow train's length to clear it.

WORKING

$$v_{rel} = 90 - 72 = 18 \text{ km/h} = 5 \text{ m/s}$$

$$\text{distance to clear} = 120 + 120 = 240 \text{ m}$$

$$t = \frac{240}{5} = \mathbf{48} \text{ s}$$

TRAP

Using only one train's length (120 m) gives 24 s. To *fully* pass, the front of A goes from level with the back of B to clear of the front of B, which is the **sum** of the lengths.

Q2 The delayed chase

JEE MAIN

A bus moving at a steady 20 m/s passes a scooter parked at a signal. The scooter starts 5 s later and chases at a steady 25 m/s. How long after the scooter starts does it catch the bus, and how far has the scooter travelled by then?

CORE PRINCIPLE

At the moment the chase begins there is a fixed head start. After that, only the relative speed closes the gap.

WORKING

Head start when the scooter starts: $20 \times 5 = 100$ m.

$$v_{rel} = 25 - 20 = 5 \text{ m/s}, \qquad t = \frac{100}{5} = \mathbf{20} \text{ s}$$

Distance covered by the scooter:

$$25 \times 20 = \mathbf{500} \text{ m}$$

TRAP

Forgetting the 5 s head start, or timing from when the bus passed instead of when the scooter started. Read the clock the question actually asks for.

Q3 Walking up a moving escalator

JEE MAIN

A man walking up a **stationary** escalator takes 90 s. Standing still on the **moving** escalator, he is carried up in 60 s. How long does he take to walk up while the escalator moves?

CORE PRINCIPLE

Velocities relative to the ground add. Work in "fraction of escalator length per second", which are just rates that sum.

WORKING

$$\text{man: } \frac{1}{90}, \qquad \text{escalator: } \frac{1}{60}$$

$$\frac{1}{90}+\frac{1}{60}=\frac{5}{180}=\frac{1}{36} \text{ per second}$$

$$t = \mathbf{36} \text{ s}$$

TRAP

Averaging the times to get 75 s. Times do not add, **rates** do. Always convert to speed or rate before combining.

2 Two-Dimensional Relative Velocity

Q4 River crossing, two goals

MAIN → ADV

A river 300 m wide flows at 4 m/s. A boat moves at 5 m/s in still water. **(a)** Cross in the shortest time: find that time and the drift. **(b)** Land at the point directly opposite: find the heading and the crossing time.

CORE PRINCIPLE

Only the component of the boat's velocity *across* the river shortens the crossing. The component *along* the river causes drift. You cannot maximise the crossing component and cancel drift with one heading.

WORKING

(a) Shortest time. The full boat speed points across:

$$t_{\min} = \frac{300}{5} = \mathbf{60}\ \text{s}, \qquad \text{drift} = 4 \times 60 = \mathbf{240}\ \text{m}$$

(b) Directly opposite. Aim upstream by angle θ from the normal so the upstream component kills the current:

$$5\sin\theta = 4 \Rightarrow \sin\theta = \tfrac{4}{5}, \quad \theta = 53^\circ$$

Across component $5\cos\theta = 5 \times \frac{3}{5} = 3\ \text{m/s}$, so:

$$t = \frac{300}{3} = \mathbf{100}\ \text{s}$$

TRAP

Believing "shortest time" and "directly opposite" are the same crossing. Shortest time gives maximum drift; zero drift costs extra time (100 s versus 60 s here).

Q5 The umbrella angle

JEE MAIN

Rain falls **vertically** at 30 m/s. A woman cycles at 10 m/s from east to west. At what angle from the vertical, and toward which side, should she hold her umbrella?

CORE PRINCIPLE

She must point the umbrella along the rain's velocity *in her frame*, which is $\vec{v}_{rain} - \vec{v}_{woman}$.

✎ DRAW THE VECTOR TRIANGLE

Put the rain vector (down, 30) and the reversed cyclist vector (east, 10) tip to tail. The hypotenuse is the apparent rain. Its lean off vertical is the umbrella angle.

WORKING

Take east as $+x$, up as $+y$. Then $\vec{v}_{rain} = (0, -30)$ and $\vec{v}_{woman} = (-10, 0)$:

$$\vec{v}_{rain,\ woman} = (0, -30) - (-10, 0) = (10, -30)$$

$$\tan\theta = \frac{10}{30} = \frac{1}{3} \Rightarrow \theta \approx \mathbf{18.4^\circ} \text{ from vertical}$$

The horizontal part points along her motion, so she tilts the umbrella **toward the direction she is riding**.

TRAP

Tilting backward into the rain. In your own frame the rain always swings **toward your direction of motion**, so the umbrella leans forward, not back.

Q6 Two cars at a right angle

JEE MAIN

Car A drives east at 30 km/h. Car B drives north at 40 km/h. Find the velocity of B relative to A (magnitude and direction).

CORE PRINCIPLE

Subtract the vectors, do not just subtract the speeds.

✎ DRAW THE VECTOR TRIANGLE

Draw $\vec{v}_B$ (north, 40) and $-\vec{v}_A$ (west, 30) tip to tail. The closing side is $\vec{v}_{BA}$, a 3-4-5 triangle pointing north-west.

WORKING

East is $+x$, north is $+y$, so $\vec{v}_A = (30, 0)$ and $\vec{v}_B = (0, 40)$:

$$\vec{v}_{BA} = (0, 40) - (30, 0) = (-30,\ 40)$$

$$|\vec{v}_{BA}| = \sqrt{30^2 + 40^2} = \mathbf{50}\ \text{km/h}$$

Direction: 30 west and 40 north, so $\tan\alpha = \frac{40}{30}$, giving $\alpha = 53^\circ$ **north of west**.

TRAP

Reporting 10 km/h (40 minus 30). Perpendicular velocities never subtract as scalars; use the vector triangle.

3 Closest Approach and Drift Optimisation

Q7 Two ships, closest approach

JEE ADVANCED

At $t=0$, ship P is at the origin moving **north** at 10 km/h. Ship Q is at position $(8,6)$ km (8 km east, 6 km north of P) moving **west** at 10 km/h. Find their minimum separation and when it occurs.

CORE PRINCIPLE

Freeze P. Then Q travels in a straight line at $\vec{v}_{rel}$. Minimum separation is the perpendicular distance from P to that line; the time comes from projecting the initial gap onto $\vec{v}_{rel}$.

WORKING

East is $+x$, north is $+y$:

$$\vec{r}_0=(8,6),\quad \vec{v}_{rel}=(-10,0)-(0,10)=(-10,-10)$$

Time of closest approach:

$$t^\star=-\frac{\vec{r}_0\cdot\vec{v}_{rel}}{|\vec{v}_{rel}|^2}=\frac{140}{200}=\mathbf{0.7}\text{ h}=\mathbf{42}\text{ min}$$

Separation vector at that instant:

$$\vec{r}(t^\star)=(8,6)+(-10,-10)(0.7)=(1,-1)$$

$$d_{\min}=\sqrt{1^2+1^2}=\sqrt{\mathbf{2}}\approx\mathbf{1.41}\text{ km}$$

TRAP

Differentiating the separation *distance* and wrestling with a square root. Minimise the squared distance $|\vec{r}_0+\vec{v}_{rel}t|^2$, a clean quadratic in t.

Q8 Least drift when the boat is slower

JEE ADVANCED

A river 500 m wide flows at 5 m/s. A boat moves at only 3 m/s in still water, so it cannot reach the point directly opposite. Find the heading that gives the **least drift**, and compute that drift and the crossing time.

CORE PRINCIPLE

With $v_{boat} < v_{river}$, zero drift is impossible, so minimise it. Write drift as a function of heading θ (measured upstream from the normal) and set the derivative to zero.

WORKING

With width d, current u, boat speed v:

$$\text{drift}(\theta) = d\left[\tfrac{u}{v}\sec\theta - \tan\theta\right]$$

Setting the derivative to zero gives the key condition:

$$\sin\theta = \tfrac{v}{u} = \tfrac{3}{5} \Rightarrow \theta = 37^\circ \text{ upstream of the normal}$$

The minimum drift simplifies to:

$$x_{\min} = \tfrac{d}{v}\sqrt{u^2 - v^2} = \tfrac{500}{3}\sqrt{25-9} = \tfrac{500}{3} \times 4 = \tfrac{\mathbf{2000}}{\mathbf{3}} \approx \mathbf{666.7}\ \text{m}$$

Crossing time, with across component $v\cos\theta = 3 \times \tfrac{4}{5} = 2.4$ m/s:

$$t = \tfrac{500}{2.4} \approx \mathbf{208.3}\ \text{s}$$

TRAP

Copying the fast-boat rule $\sin\theta = u/v$. That needs $v > u$ and gives zero drift. When $v < u$ the optimisation flips to $\sin\theta = v/u$. Check which speed is larger before choosing the formula.

4 Relative Motion with Equal Accelerations

Q9 Two balls, one straight line

JEE ADVANCED

From the top of a tower, two balls are thrown at the same instant in the same vertical plane: ball 1 horizontally at 10 m/s, ball 2 at 45° above the horizontal at $10\sqrt{2}$ m/s (same horizontal direction). Describe the motion of ball 2 as seen from ball 1, and find their separation at $t = 2$ s.

CORE PRINCIPLE

Both balls share acceleration $\vec{g}$, so $\vec{a}_{rel} = 0$. Relative velocity stays constant, so the relative path is a **straight line** at uniform speed. Gravity vanishes in the relative frame.

WORKING

$$\vec{v}_1 = (10,\ 0), \quad \vec{v}_2 = (10\sqrt{2}\cos 45^\circ,\ 10\sqrt{2}\sin 45^\circ) = (10,\ 10)$$

$$\vec{v}_{21} = \vec{v}_2 - \vec{v}_1 = (0,\ 10)$$

So from ball 1, ball 2 moves straight up at a constant 10 m/s:

$$\text{separation} = |\vec{v}_{21}|\, t = 10 \times 2 = \mathbf{20}\ \text{m, vertical}$$

TRAP

Adding a $\frac{1}{2}gt^2$ term to the relative motion. In the relative frame the accelerations cancel, so the separation grows **linearly**, not quadratically.

Q10 Rain that changes with your speed

JEE ADVANCED

A man walking due east at 3 m/s finds the rain falling **vertically**. When he speeds up to 6 m/s (still due east), the rain appears to come at 45° to the vertical. Find the true speed and direction of the rain.

CORE PRINCIPLE

The rain's true velocity is fixed; two apparent observations give two equations. **Set the axes up front:** let $+x$ be east and $-y$ be downward, and write the rain as $(a,\ -b)$ with $b>0$.

WORKING

Condition 1 (at 3 m/s the rain looks vertical, so the horizontal part of the relative velocity is zero):

$$a-3=0 \Rightarrow a=3 \text{ m/s}$$

Condition 2 (at 6 m/s the relative horizontal is $3-6=-3$; a 45° tilt needs equal horizontal and vertical magnitudes):

$$\frac{|-3|}{b}=\tan 45^\circ=1 \Rightarrow b=3 \text{ m/s}$$

True rain velocity $(3,\ -3)$:

$$\text{speed}=\sqrt{3^2+3^2}=\mathbf{3\sqrt{2}}\approx\mathbf{4.24}\text{ m/s},\ \mathbf{45^\circ}\ \text{from vertical}$$

TRAP

Assuming the rain is vertical because case 1 "looks" vertical. Vertical *to the moving man* is not vertical to the ground. The rain genuinely has a horizontal component; the 3 m/s walk just cancels it in case 1.

One-Line Revision Recap

The single move that unlocks each situation. Print this page alone.

Situation	Key move
Overtake or cross	relative displacement = sum of lengths
Shortest crossing time	point straight across, accept the drift
Zero drift, $v_{boat} > v_{river}$	aim upstream, $\sin\theta = u/v$
Least drift, $v_{boat} < v_{river}$	aim upstream, $\sin\theta = v/u$
Rain and man	tilt umbrella along $\vec{v}_{rain} - \vec{v}_{man}$
Closest approach	perpendicular distance to the $\vec{v}_{rel}$ line
Two projectiles	$\vec{a}_{rel} = 0$, relative path is a straight line

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★ CONCEPT STILL NOT CLICKING?

Play it. Then practice it.

If a problem here didn't land, open the topic in the Logic Bloom app: play the concept as a game, then drill practice questions till it clicks.

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