LOGIC BLOOM · JEE PHYSICS

Motion in a Straight Line *& in a Plane*

Class 11 Physics · **Kinematics** · JEE Main & Advanced

Complete notes with a deep dive into relative velocity: rain-umbrella, boat-and-river, crossing the river, and pursuit problems.

Theory | Diagrams | Relative Velocity Applications | Formula Sheet

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Contents

PART I

Motion in a Straight Line

One-dimensional kinematics: the definitions, equations and graphs that everything else builds on.

1 Basic Terms: Distance, Displacement, Speed, Velocity

Motion in a straight line (rectilinear motion) is the simplest kind of motion, yet its definitions underpin the whole of mechanics. The first thing JEE tests is whether you keep the scalar and vector quantities apart.

Quantity	Type	Meaning
Distance	Scalar	Total path length covered. Never decreases, never negative.
Displacement	Vector	Straight-line change in position (final − initial). Can be zero or negative.
Speed	Scalar	Distance / time. Always ≥ 0.
Velocity	Vector	Displacement / time. Sign gives direction.
Acceleration	Vector	Rate of change of velocity.

AVERAGE VS INSTANTANEOUS

Average velocity = total displacement / total time; average speed = total distance / total time. These are equal only when motion is along a straight line without reversing direction.

Instantaneous velocity is the limit as the time interval shrinks to zero: $v = dx/dt$. Instantaneous speed is its magnitude. The slope of the position-time graph at a point gives the instantaneous velocity there.

EXAMPLE · AVERAGE SPEED VS AVERAGE VELOCITY

A body goes 60 m east in 20 s, then 20 m west in 10 s. Total distance = 80 m, total time = 30 s, so average speed = 80/30 ≈ 2.67 m/s. Net displacement = 60 − 20 = 40 m east, so average velocity = 40/30 ≈ 1.33 m/s east. They differ because the motion reversed.

2 The Equations of Motion (Uniform Acceleration)

When acceleration a is constant, three equations connect the five quantities u (initial velocity), v (final velocity), a, t (time) and s (displacement). Each equation leaves out one variable, so you pick the one missing the quantity you neither know nor want.

```
v = u + at   (no s)
s = ut + ½at²   (no v)
v² = u² + 2as   (no t)
distance in the n-th second: sₙ = u + ½a(2n - 1)
```

⚠ **SIGN CONVENTION IS EVERYTHING**

Fix a positive direction first, then give every vector (u, v, a, s) its correct sign. For a body thrown up, take up as positive: u is +, but $a = -g = -9.8$ m/s^2. Mixing signs is the single biggest source of errors in this chapter.

EXAMPLE · FREE FALL

A ball is thrown upward at $u = 20$ m/s (take $g = 10$ m/s^2, up positive so $a = -10$). Find the maximum height and time to reach it.

At the top $v = 0$. Using $v = u + at$: $0 = 20 - 10t$, so $t = 2$ s. Using $v^2 = u^2 + 2as$: $0 = 400 - 20s$, so $s = 20$ m. The ball rises 20 m in 2 s.

EXAMPLE · DISTANCE IN THE N-TH SECOND

A car starts from rest with $a = 2$ m/s^2. Distance covered in the 3rd second: $s_3 = u + ½a(2n-1) = 0 + ½(2)(2×3-1) = 5$ m. Note this is a distance during a one-second interval, not a total displacement.

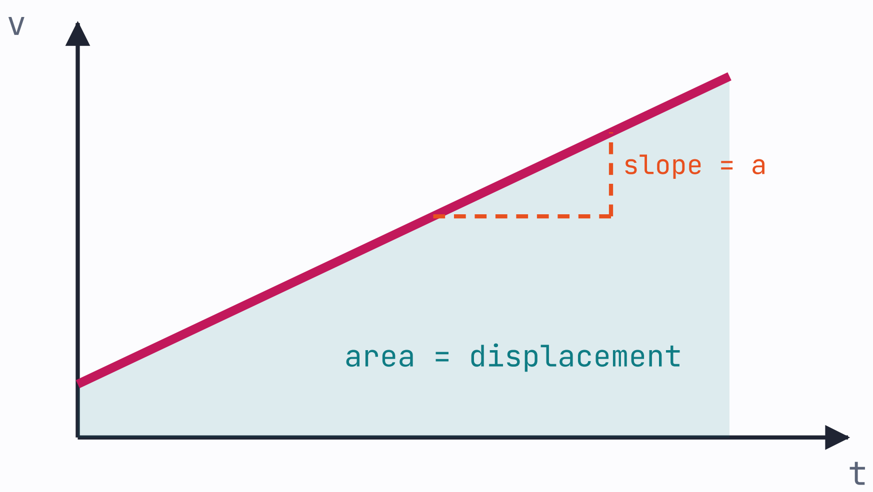
3 Graphs of Motion

Graphs turn a word problem into a picture, and JEE loves extracting information from slopes and areas. Two graph types matter most.

READING THE GRAPHS

Position-time (x-t): the slope at any point is the velocity. A straight line means uniform velocity; a curve means acceleration. A horizontal line means the body is at rest.

Velocity-time (v-t): the slope is the acceleration, and the area under the graph is the displacement. This area rule is one of the most useful shortcuts in kinematics.



On a velocity-time graph the slope of the line gives the acceleration, and the shaded area under it gives the displacement. A straight line means uniform acceleration.

EXAMPLE · AREA UNDER V-T GRAPH

A body accelerates uniformly from rest to 20 m/s in 5 s. The v-t graph is a straight line from (0,0) to (5,20). Displacement = area of the triangle = ½ × base × height = ½ × 5 × 20 = 50 m. Acceleration = slope = 20/5 = 4 m/s^2.

PART II

Motion in a Plane

Two-dimensional motion with vectors and projectiles, where the two axes act independently.

4 Vectors: The Language of Plane Motion

Motion in a plane is two-dimensional, so every quantity (position, velocity, acceleration) becomes a vector with two components. The key simplification for JEE is that the two perpendicular directions are independent: x-motion and y-motion do not affect each other.

A vector: $\mathbf{A} = A_x\,\hat{i} + A_k\,\hat{j}$ · magnitude $|A| = \sqrt{(A_x^2 + A_k^2)}$

direction: $\tan\theta = A_k / A_x$

addition: $\mathbf{A} + \mathbf{B} = (A_x+B_x)\hat{i} + (A_k+B_k)\hat{j}$

RESOLVING INTO COMPONENTS

A vector of magnitude A at angle θ to the x-axis splits into $A_x = A\cos\theta$ (horizontal) and $A_k = A\sin\theta$ (vertical). Working with components turns vector problems into two separate scalar problems, one per axis, which you solve with the 1D equations above.

EXAMPLE · ADDING TWO VELOCITIES

A particle has velocity $3\hat{i} + 4\hat{j}$ m/s. Its speed is $\sqrt{(3^2 + 4^2)} = 5$ m/s, and its direction makes $\tan\theta = 4/3$, so $\theta \approx 53°$ above the x-axis. Components in, magnitude and angle out.

5 Projectile Motion

A projectile is the classic plane-motion problem: horizontal velocity stays constant (no horizontal force) while vertical motion is free fall under gravity. Treat the two axes independently and stitch the answers together.

```
For launch speed υ at angle θ (g downward):
time of flight T = 2υ sinθ / g
maximum height H = υ²sin²θ / 2g
horizontal range R = υ²sin2θ / g  (max at θ = 45°)
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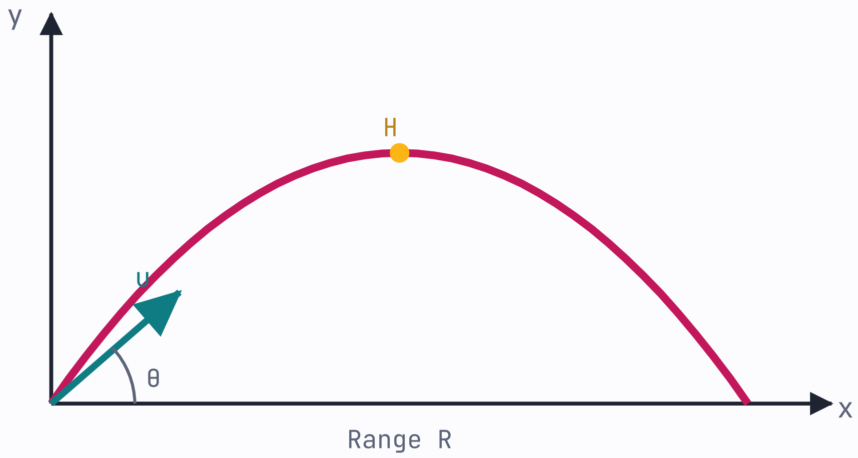


A projectile follows a parabola. The horizontal velocity u cosθ is constant; the vertical velocity changes under gravity, becoming zero at the peak height H. The range R is the horizontal distance covered.

EXAMPLE · PROJECTILE

A ball is launched at $u = 20$ m/s, $\theta = 30°$ ($g = 10$). Then $\sin 30° = 0.5$, $\sin 60° = 0.866$. Time of flight $T = 2(20)(0.5)/10 = 2$ s; height $H = (20^2)(0.5^2)/(2\times10) = 5$ m; range $R = (20^2)(0.866)/10 \approx 34.6$ m. Maximum range for a given speed always occurs at 45°.

PART III

Relative Velocity & Applications

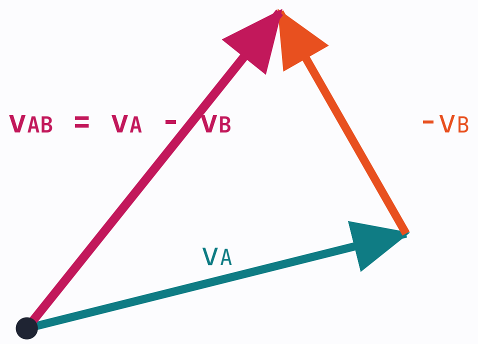
The rain-umbrella, boat-and-river and pursuit problems, each solved from one simple vector idea.

6 Relative Velocity: The Core Idea

Relative velocity is the velocity of one body as measured by an observer moving with another. It is the single most important tool for the rain, boat and pursuit problems, and it is pure vector subtraction.

velocity of A relative to B: $v_{AB} = v_A - v_B$

and $v_{BA} = -v_{AB}$ · $|v_{AB}| = |v_{BA}|$



velocity of A relative to B: subtract vB (add -vB)

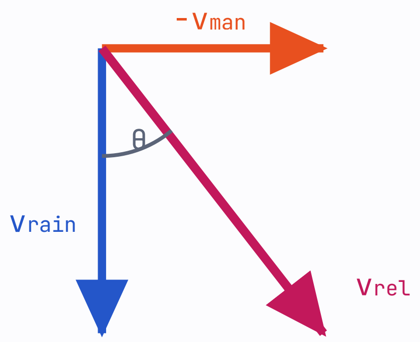
To get the velocity of A relative to B, reverse v_B and add it to v_A. The closing vector of the triangle is v_AB. Everything in this section is an application of this one rule.

HOW TO SET UP ANY RELATIVE-VELOCITY PROBLEM

1. Write each real velocity as a vector (choose axes, usually east and north, or across and along).
2. Form the relative velocity by subtracting the observer's velocity.
3. Read off magnitude and direction, or set a component to zero if the problem asks for a special condition (zero drift, rain appearing vertical, and so on).

7 Application 1: Rain and Umbrella

A person walking in vertically falling rain must tilt the umbrella forward. Why? Because in the person's own frame the rain no longer falls straight down: it acquires a backward horizontal velocity equal and opposite to the walking velocity.



tanθ = v_{man} / v_{rain} · tilt umbrella by θ into motion

In the walker's frame, rain velocity relative to the man is v_rain (down) plus (−v_man) (backward). The resultant is tilted, so the umbrella must point along it, at angle θ from the vertical.

$$v_{rain,man} = v_{rain} - v_{man}$$

angle from vertical: $\tan\theta = v_{man} / v_{rain}$

apparent speed = $\sqrt{v_{rain}^2 + v_{man}^2}$

EXAMPLE · TILT THE UMBRELLA

Rain falls vertically at 10 m/s; a man walks horizontally at 5 m/s. In his frame the rain has a backward horizontal component of 5 m/s and a downward component of 10 m/s. The umbrella must tilt into the direction of motion at $\tan\theta = 5/10 = 0.5$, so $\theta \approx 26.6°$ from the vertical. Apparent rain speed $= \sqrt{(10^2+5^2)} \approx 11.2$ m/s.

▲ THE 'RAIN APPEARS VERTICAL' TWIST

Sometimes the rain itself falls at an angle (it has a horizontal velocity). If a man walks so that the rain appears to fall vertically, his velocity must exactly match the horizontal component of the rain's velocity. Example: rain velocity = $3\hat{i} - 4\hat{j}$. If the man walks at $3\hat{i}$, the relative velocity is $(3-3)\hat{i} - 4\hat{j} = -4\hat{j}$, purely vertical. So he must walk at 3 m/s in the +x direction.

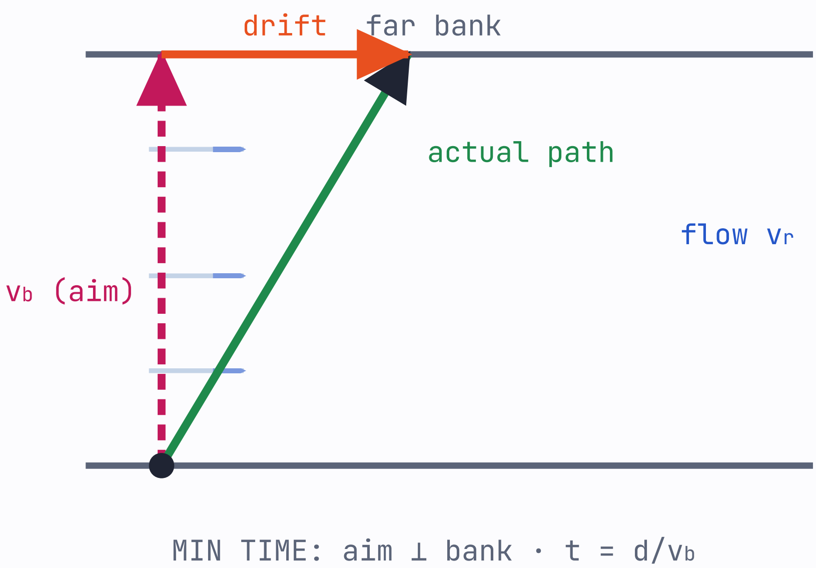
8 Application 2: Boat and River (Crossing the River)

A boat that can move at speed v_b relative to the water must cross a river whose current flows at v_r. There are two standard goals, and they give different aiming directions. Keep them separate.

CASE A: CROSS IN MINIMUM TIME

To cross in the least time, aim the boat straight across (perpendicular to the bank). Then the whole of v_b works on crossing, and the current only carries you downstream.

time $t = d / v_b$ · drift (downstream) $= v_r \times t = v_r d / v_b$ · resultant speed $= \sqrt{v_b^2 + v_r^2}$.



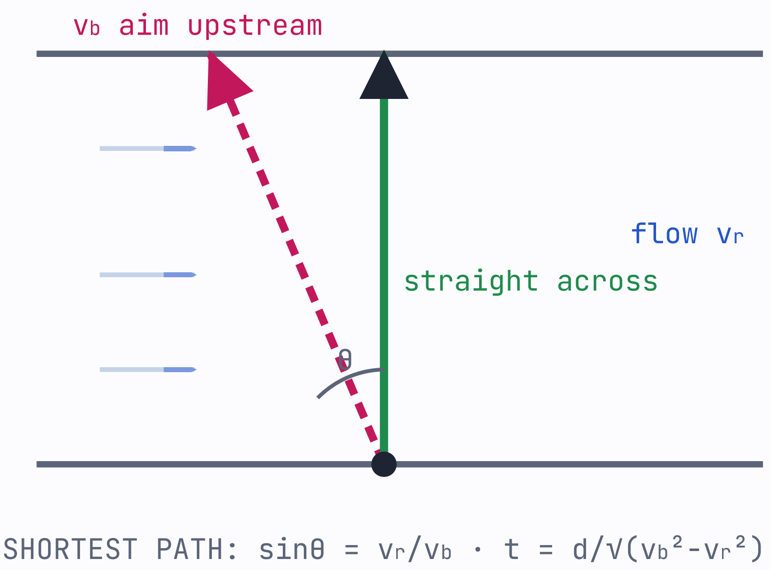
Minimum time: the boat is aimed perpendicular to the bank. It crosses in $t = d/v_b$, but the current sweeps it downstream by a drift of $v_r \cdot t$. The actual path is the diagonal.

CASE B: CROSS BY THE SHORTEST PATH (ZERO DRIFT)

To land directly opposite the start (no drift), aim the boat upstream at an angle θ so the upstream component of v_b cancels the current: $v_b \sin\theta = v_r$.

$\sin\theta = v_r / v_b$ · effective speed across $= \sqrt{v_b^2 - v_r^2}$ · time $t = d / \sqrt{v_b^2 - v_r^2}$.

This is only possible if $v_b > v_r$. The shortest path takes longer than the minimum-time crossing.



Shortest path: the boat is aimed upstream at angle θ with sinθ = v_r/v_b, so the current is exactly cancelled and the boat travels straight across with zero drift.

EXAMPLE · BOTH CASES, ONE RIVER

River width d = 100 m, boat v_b = 5 m/s, current v_r = 3 m/s.

Minimum time: aim across, t = 100/5 = 20 s, drift = 3 × 20 = 60 m downstream.

Shortest path: aim upstream at sinθ = 3/5, so θ ≈ 36.9°. Speed across = $\sqrt{(5^2-3^2)}$ = 4 m/s, time = 100/4 = 25 s, drift = 0. The zero-drift crossing takes 5 s longer.

▲ WHEN THE CURRENT IS FASTER THAN THE BOAT (V_R > V_B)

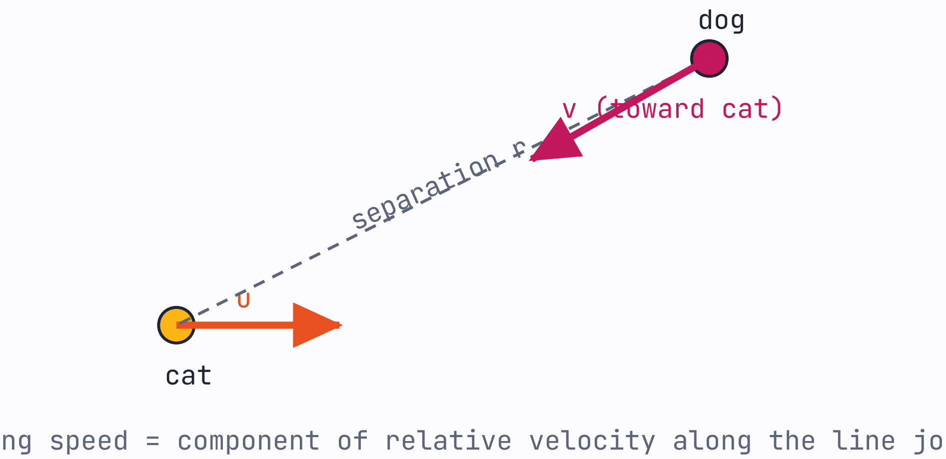
Zero drift is now impossible, so the goal becomes minimum drift. Aim the boat upstream at the angle θ (measured from the straight-across direction) given by sinθ = v_b/v_r. Minimising the drift by setting its derivative to zero gives:

```
minimum drift = d √(v_r² - v_b²) / v_b   (at sinθ = v_b/v_r)
```

For v_b = 3, v_r = 5, d = 100: minimum drift = 100√(25−9)/3 = 100(4)/3 ≈ 133 m. This optimisation is a common JEE Advanced extension.

9 Application 3: Dog and Cat (Pursuit and Closing Speed)

Pursuit problems ask when a chaser meets a target, or how fast the gap between two bodies shrinks. The trick is the closing speed: the rate at which the separation decreases equals the component of the relative velocity along the line joining the two bodies.



The dog chases the cat. The gap between them shrinks at a rate equal to the component of the relative velocity along the line joining them: this is the closing speed.

```
closing speed = component of (v_chaser - v_target) along the line joining
time to meet = initial separation / closing speed  (when closing speed is constant)
```

EXAMPLE · HEAD-ON CLOSING SPEED

A dog at x = 0 runs in the +x direction at 4 m/s; a cat at x = 12 m runs toward the dog at 2 m/s. They move directly toward each other, so the closing speed = 4 + 2 = 6 m/s. Time to meet = 12 / 6 = 2 s.

EXAMPLE · FOUR BODIES ON A SQUARE (CLASSIC)

Four insects sit at the corners of a square of side a = 10 m. Each moves at speed v = 2 m/s always directed toward the next insect (anticlockwise). By symmetry they spiral into the centre. At every instant a target moves perpendicular to the line joining it to its chaser, so it neither approaches nor recedes along that line: the closing speed is just v. Time to meet = a / v = 10 / 2 = 5 s.

★ THE ONE INSIGHT BEHIND EVERY PURSUIT PROBLEM

Only the component of relative velocity along the line joining the two bodies changes their separation. The perpendicular component merely rotates that line. Resolve the relative velocity into these two directions and the problem collapses to a one-dimensional closing-speed calculation.

PART IV

For JEE Advanced

Variable-acceleration calculus, inclined projectiles, graph transformations and general minimum approach: the rigorous layers Advanced demands.

10 Variable (Non-Uniform) Acceleration

The equations $v = u + at$ and their siblings apply only when acceleration is constant. JEE Advanced routinely gives acceleration as a function of time, velocity or position, and then you must go back to calculus. Everything follows from the definitions $v = dx/dt$ and $a = dv/dt$.

★ THE THREE DIFFERENTIAL FORMS YOU MUST KNOW

Which form to use depends on what the acceleration is given in terms of:

- a as a function of time, $a = f(t)$: integrate a dt to get v, then integrate v dt to get x.
- a as a function of velocity, $a = f(v)$: use $a = dv/dt$, so $dt = dv/f(v)$, and integrate.
- a as a function of position, $a = f(x)$: use the chain-rule form **$a = v\,dv/dx$**, so $v\,dv = f(x)\,dx$.

```
v = dx/dt  ·  a = dv/dt  ·  a = v dv/dx
a = f(t): v = ∫ a dt  ·  a = f(v): t = ∫ dv/a  ·  a = f(x): ∫ v dv = ∫ a dx
```

EXAMPLE · ACCELERATION DEPENDS ON VELOCITY

A particle starts at u and decelerates as $a = -kv$ (resistive motion). Find v(t) and the total distance.

Since $a = dv/dt = -kv$, separate variables: $dv/v = -k\,dt$. Integrating from u to v and 0 to t gives $\ln(v/u) = -kt$, so **$v = u\,e^{-kt}$**. The velocity decays exponentially and never quite reaches zero.

For total distance, use $x = \int v\,dt = \int_0^\infty u\,e^{-kt}\,dt =$ **u/k**. A finite distance despite never stopping.

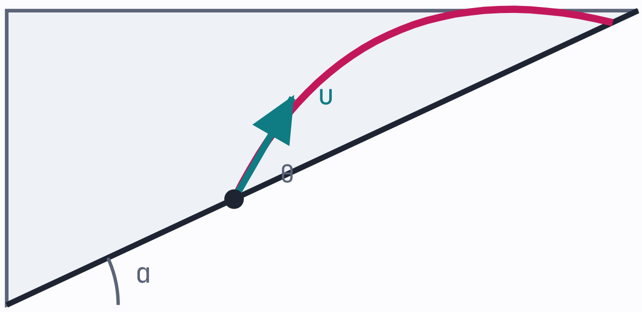
EXAMPLE · ACCELERATION DEPENDS ON POSITION

A particle moves with $a = -\omega^2 x$ (this is simple harmonic motion). Find v in terms of x, given speed u at $x = 0$.

Use $a = v\,dv/dx = -\omega^2 x$, so $v\,dv = -\omega^2 x\,dx$. Integrating: $v^2/2 = u^2/2 - \omega^2x^2/2$, giving **$v^2 = u^2 - \omega^2x^2$**. This is the standard velocity-position relation for SHM.

11 Projectile Motion on an Inclined Plane

Launching a projectile up or down a slope is a staple Advanced profile. The trick is to tilt your axes along and perpendicular to the incline, so gravity splits into two components instead of one.



g splits: g sinα along incline, g cosα perpendicular

On an incline of angle α, take axes along and perpendicular to the surface. Gravity resolves into g sinα along the incline (down-slope) and g cosα perpendicular to it. The projectile is launched at angle θ to the incline.

For projection speed u at angle θ to an incline of angle α (projected up the slope):

time of flight $T = 2u \sin\theta / (g \cos\alpha)$

range up the incline $R = 2u^2 \sin\theta \cos(\theta+\alpha) / (g \cos^2\alpha)$

maximum range up the incline: $R_{max} = u^2 / [g(1 + \sin\alpha)]$ at $\theta = 45° - \alpha/2$

▲ UP THE SLOPE VS DOWN THE SLOPE

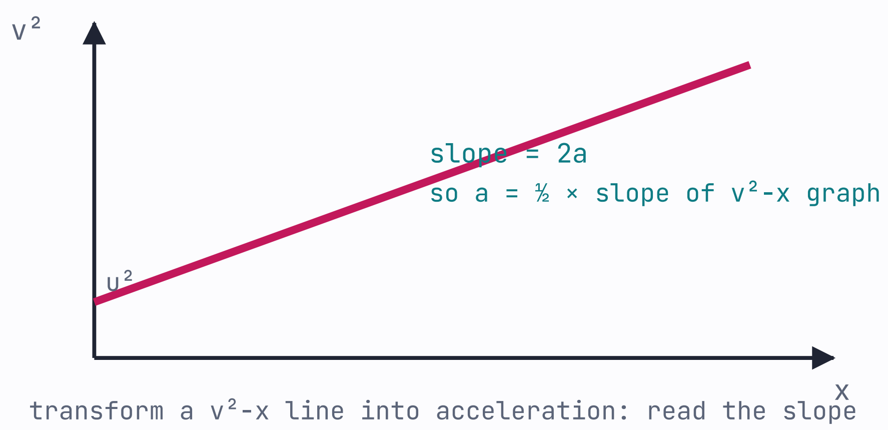
For projection down the incline, flip the sign of α: the down-slope component of gravity now aids the motion. The maximum range down the incline is $R_{max} = u^2 / [g(1 - \sin\alpha)]$, which is larger than the up-slope range. The optimum angle also shifts to $\theta = 45° + \alpha/2$.

EXAMPLE · PERPENDICULAR LANDING CONDITION

A common Advanced twist asks when the projectile lands perpendicular to the incline. That happens when the velocity component along the incline becomes zero at landing: $u \cos\theta = g \sin\alpha \cdot T$. Setting up the two-axis equations and imposing this condition pins down the required angle. Always resolve gravity first, then apply the 1D equations on each tilted axis.

12 Advanced Graph Transformations

Main-level questions ask you to read a slope or an area. Advanced asks you to convert one kind of graph into another, for example turning a v-x or v^2-x graph into an a-x graph. The bridge is always a = v dv/dx.



A v^2-x graph that is a straight line has constant slope. Since $v^2 = u^2 + 2ax$, the slope of the v^2-x line equals 2a, so the acceleration is half the slope: a straight v^2-x line means uniform acceleration.

HOW TO CONVERT BETWEEN GRAPHS

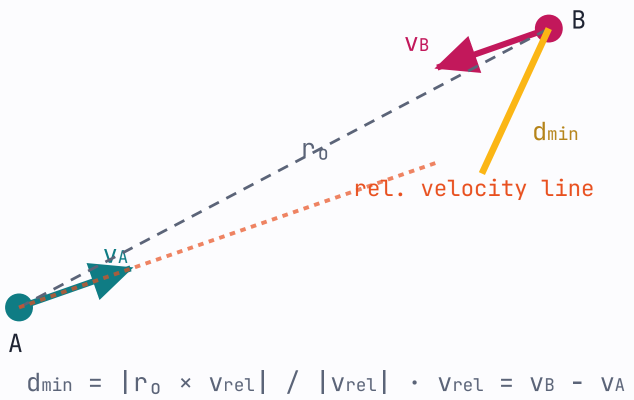
- From a **v-x** graph: read v and the slope dv/dx at a point, then a = v (dv/dx). A straight v-x line does *not* mean constant acceleration, since a depends on v as well.
- From a **v^2-x** graph: the slope is $d(v^2)/dx = 2a$, so a = ½ × slope. A straight line here *does* mean constant acceleration.
- From an **a-x** graph: the area under it equals $\int a\,dx = \int v\,dv = \frac{1}{2}(v^2 - u^2)$, which gives the change in v^2.

EXAMPLE · READING A FROM A V-X LINE

A particle has a velocity-position graph that is a straight line from (x=0, v=10) to (x=20, v=0), so v = 10 − ½x. Then dv/dx = −½. At x = 0, v = 10, so a = v(dv/dx) = 10(−½) = −5 m/s². At x = 20, v = 0, so a = 0. The acceleration is *not* constant even though the v-x graph is a straight line.

13 General Minimum Distance of Approach

The insect-on-a-square problem uses symmetry. The general Advanced problem gives two particles on arbitrary straight lines at different speeds and asks for the closest they ever get. The clean way is relative velocity: freeze one particle and watch the other move in a straight line.



Sit in A's frame: B moves along a straight line with velocity v_rel = v_B − v_A. The closest approach is the perpendicular distance from A to that line, so d_min = |r_0 × v_rel| / |v_rel|.

$$\text{relative velocity } v_{rel} = v_B - v_A$$
$$\text{time of closest approach } t^* = -(r_0 \cdot v_{rel}) / |v_{rel}|^2$$
$$\text{minimum distance } d_{min} = |r_0 \times v_{rel}| / |v_{rel}|$$

★ THE METHOD IN THREE STEPS

1. Find the relative velocity v_rel = v_B − v_A and the initial separation vector r_0 (from A to B).
2. In A's frame, B travels in a straight line along v_rel. The minimum distance is the perpendicular distance from A to that line.
3. Compute it as the magnitude of the cross product |r_0 × v_rel| divided by |v_rel|. If t* comes out negative, the closest approach was in the past and the particles are already separating.

EXAMPLE · TWO PARTICLES, ARBITRARY LINES

Particle A is at (10, 0) moving at (0, 2) m/s; particle B is at the origin moving at (−3, 0)... treat A as the reference. Here r_0 (from A to B) = (−10, 0) and v_rel = v_B − v_A = (−3, −2). Then |v_rel| = √13, the cross product r_0 × v_rel = (−10)(−2) − (0)(−3) = 20, so d_min = 20/√13 ≈ 5.55 m. The whole 2D chase collapses to one perpendicular-distance calculation.

★ Motion: One-Page Formula Sheet

Everything for revision day. Print this alone.

1D EQUATIONS

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$s_n = u + \frac{1}{2}a(2n-1)$

GRAPHS

x-t slope = velocity

v-t slope = acceleration

v-t area = displacement

straight x-t = uniform v

VECTORS

$A = A_x\hat{i} + A_y\hat{j}$

$|A| = \sqrt{A_x^2+A_y^2}$

$A_x = A\cos\theta$, $A_y = A\sin\theta$

$\tan\theta = A_y/A_x$

PROJECTILE

$T = 2u\sin\theta/g$

$H = u^2\sin^2\theta/2g$

$R = u^2\sin 2\theta/g$

R max at $\theta=45°$

RELATIVE VELOCITY

$v_{AB} = v_A - v_B$

$v_{BA} = -v_{AB}$

subtract observer's v

work in components

RAIN-UMBRELLA

$v_{rain,man} = v_{rain} - v_{man}$

$\tan\theta = v_{man}/v_{rain}$

tilt umbrella by θ

appears vertical: match horiz comp

BOAT MIN TIME

aim ⊥ bank

$t = d/v_b$

drift $= v_r\, d/v_b$

speed $= \sqrt{v_b^2+v_r^2}$

BOAT SHORTEST PATH

aim upstream, $\sin\theta = v_r/v_b$

zero drift

v across $= \sqrt{v_b^2-v_r^2}$

$t = d/\sqrt{v_b^2-v_r^2}$

PURSUIT

closing speed along join

t = separation/closing speed

square: $t = a/v$

⊥ comp only rotates line

VARIABLE ACCEL

$a = dv/dt = v\, dv/dx$

$a=f(t)$: $v=\int a\, dt$

$a=f(v)$: $t=\int dv/a$

$a=f(x)$: $\int v\, dv=\int a\, dx$

INCLINE PROJECTILE

axes along/⊥ incline

g→ $g\sin\alpha$, $g\cos\alpha$

$T = 2u\sin\theta/(g\cos\alpha)$

R max up $= u^2/g(1+\sin\alpha)$

R max down $= u^2/g(1-\sin\alpha)$

GRAPH TRANSFORMS

v-x: $a = v(dv/dx)$

v^2-x slope = 2a

$a = \frac{1}{2} \times$ slope(v^2-x)

area under a-x $= \frac{1}{2}\Delta(v^2)$

MIN APPROACH

$v_{rel} = v_B - v_A$

$t^* = -(r_0\cdot v_{rel})/|v_{rel}|^2$

$d_{min} = |r_0\times v_{rel}|/|v_{rel}|$

freeze A, B goes straight

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Understand through games. Score through practice.

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