LOGIC BLOOM · PHYSICS · MECHANICS

Circular Motion

Banking · Radius of Curvature · **JEE Advanced depth** · NEET sections marked

Companion to the Friction and Block Systems set. Every circular problem reduces to one question: which real force is pointing at the centre? Banking, vertical circles, the rotor and the turntable all follow from it, and the hardest of them are friction problems in disguise.

7 Sections · 12 Diagrams · 14 Worked Questions · NEET Marked · Fact Sheet

Contents

0 How to Read This Set

Second of a pair. The **Friction and Block Systems** set built the idea of limiting friction; this one spends it. Banking with friction, the turntable and the rotor all use the same ceiling μN, only now the friction is pointing at the centre of a circle.

IN NEET AND JEE	Angular kinematics, centripetal force, conical pendulum, banking of roads.
JEE MAIN AND ADVANCED	Radius of curvature, the vertical circle, turntable and rotor problems.
JEE ADVANCED MAINLY	Where the string goes slack, sliding off a dome, curvature from vectors.

THINK IT THROUGH

THE ONE QUESTION TO ASK, EVERY SINGLE TIME

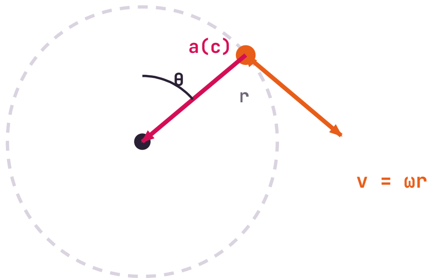
There is no such thing as a centripetal force in a free-body diagram. Centripetal is a **job**, and some ordinary force does it: tension, friction, gravity, or a normal reaction. Identify that force first and write it equal to mv^2/r. Every problem in this set is that one sentence, applied carefully.

1 Angular Kinematics

IN NEET AND JEE

Describing a rotating body by angle rather than distance turns awkward geometry into ordinary algebra, and every angular quantity has a linear twin.

ANGULAR AND LINEAR QUANTITIES ARE THE SAME MOTION, DESCRIBED TWO WAYS



$\omega = d\theta/dt$ and $\alpha = d\omega/dt$
$v = \omega r$, tangential $a(t) = \alpha r$
centripetal $a(c) = v^2/r = \omega^2 r$
one revolution: $T = 2\pi/\omega$, $f = 1/T$

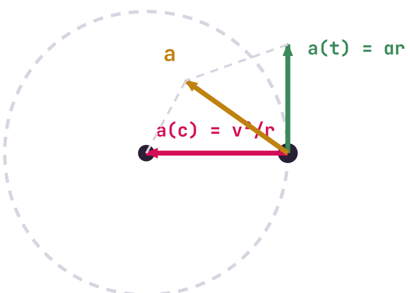
a(c) always points at the CENTRE and changes only the DIRECTION of v.
a(t) is along v and changes its SIZE.

Angular and linear descriptions of the same motion. The radius is the exchange rate between them.

★ THE DICTIONARY

$$v = \omega r, \qquad a_t = \alpha r, \qquad a_c = \frac{v^2}{r} = \omega^2 r$$
$$\omega = \frac{2\pi}{T} = 2\pi f$$

WHEN THE SPEED IS ALSO CHANGING, ACCELERATION IS NOT JUST CENTRIPETAL



$|a| = \sqrt{a(c)^2 + a(t)^2}$

and it leans off the radius by
tan(angle) = a(t) / a(c)

Uniform circular motion is the special case a(t) = 0. Speed constant, velocity still changing.

If the speed is changing too, the total acceleration leans off the radius. Only in uniform circular motion does acceleration point exactly at the centre.

TRAP ALERT

⚠ UNIFORM CIRCULAR MOTION IS NOT UNACCELERATED MOTION

Constant **speed** is not constant velocity. The direction is changing every instant, so there is an acceleration of v^2/r pointing at the centre the whole time. Questions phrased as constant speed are inviting you to answer zero acceleration.

▶ PLAY

ANGULAR QUANTITIES NOT CLICKING?

Spin it yourself and watch v and a change together.

Drag the angular speed and see the velocity arrow lengthen while the centripetal arrow grows as the square. The ω^2 in the formula stops being something you take on trust. **logicbloom.co.in/download** · Free to start.

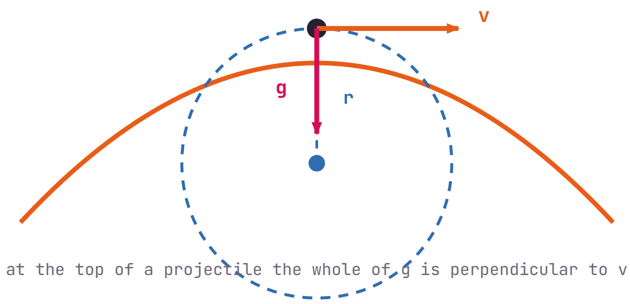
SCAN

2 Radius of Curvature

JEE MAIN AND ADVANCED

Any curved path, even one that is not a circle, is momentarily copying some circle. The radius of that circle is what turns projectile questions into circular-motion questions.

RADIUS OF CURVATURE: the circle the path is momentarily copying



r = v² / a(perp)

a(perp) is the part of the acceleration at right angles to the velocity

At the top: a(perp) = g, so r = v²/g
At launch: a(perp) = g cosθ,
so r = u²/(g cosθ)

At the top of a projectile the whole of gravity is perpendicular to the velocity, so the path is copying a circle of radius v^2/g there.

★ **CURVATURE FROM THE PERPENDICULAR ACCELERATION**

$$r = \frac{v^2}{a_\perp}, \qquad a_\perp = \frac{|\vec{v} \times \vec{a}|}{|\vec{v}|}$$

Only the part of the acceleration at right angles to v bends the path. The parallel part just changes speed.

THINK IT THROUGH

WHY THE TOP OF A PROJECTILE IS THE EASY CASE

At the highest point the velocity is horizontal and gravity is vertical, so the two are already perpendicular and a_{perp} is simply g. Anywhere else you must first resolve g perpendicular to the velocity, which gives g cos θ at the launch point.

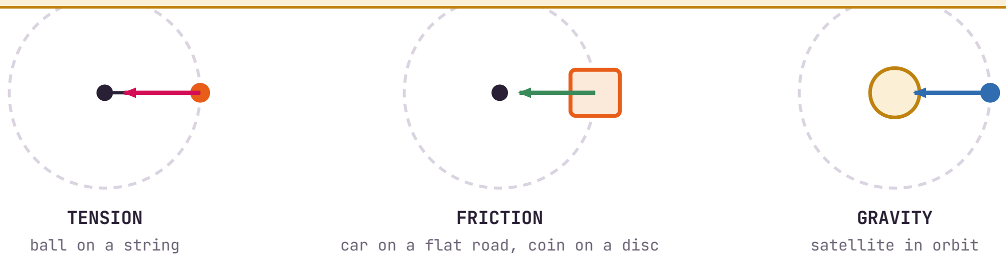
3 What Supplies the Centripetal Force

IN NEET AND JEE

This section is short because the idea is short, and it is the whole chapter.

CENTRIPETAL FORCE IS A ROLE, NOT A NEW KIND OF FORCE

Always ask WHICH real force points at the centre. Never add a separate centripetal force to a diagram.



Three different circular motions, three completely different forces doing the same job. Nothing here is a new kind of force.

★ **THE MASTER EQUATION**

$$F_{net,\ towards\ centre} = \frac{mv^2}{r} = m\omega^2 r$$

MYTH CHECK

MYTH

A centrifugal force throws you outward when a car turns, and it is the Newton's third law reaction to the centripetal force.

THE SCIENCE

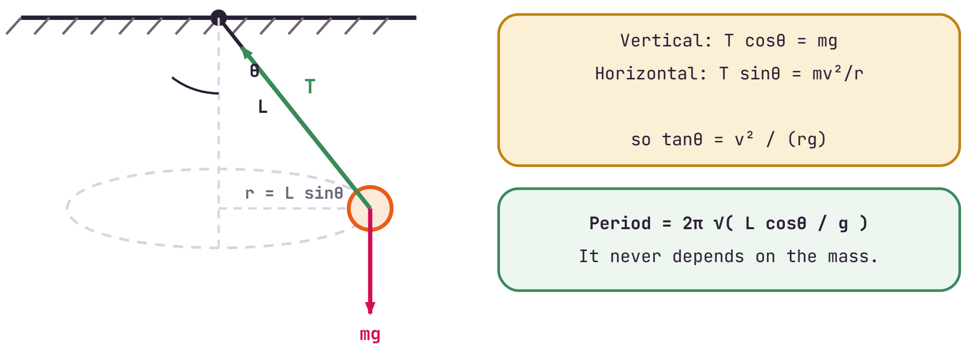
Both halves are wrong. Centrifugal force is a **pseudo-force** that exists only in the rotating frame, and a third-law pair must act on **two different bodies**. The reaction to the road's inward friction on the tyres is the tyres' outward friction on the road, not anything acting on the passenger.

4 The Conical Pendulum

IN NEET AND JEE

A bob swung so that the string sweeps out a cone. It is the cleanest possible example of resolving one force into a vertical balance and a horizontal centripetal part.

CONICAL PENDULUM: the bob circles, the string sweeps a cone



The tension does two jobs at once: its vertical component holds the bob up, its horizontal component turns it.

★ CONICAL PENDULUM

$$T\cos\theta = mg, \qquad T\sin\theta = \frac{mv^2}{r}, \qquad r = L\sin\theta$$

$$\tan\theta = \frac{v^2}{rg}, \qquad \text{Period} = 2\pi\sqrt{\frac{L\cos\theta}{g}}$$

TRAP ALERT

⚠ THE TENSION IS NEVER MG

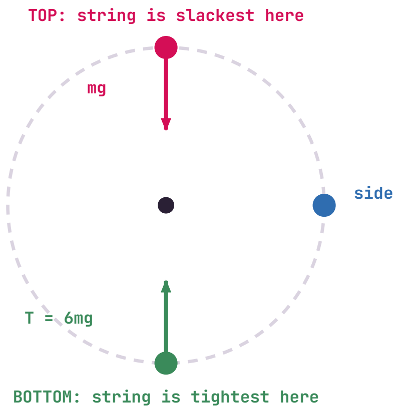
Since T cos θ = mg, the tension is mg/cos θ, which is always **larger** than the weight and grows without limit as the string approaches horizontal. That is why a string can never be pulled perfectly flat, no matter how fast the bob is swung.

5 The Vertical Circle

JEE MAIN AND ADVANCED

Speed now varies around the loop, because gravity helps on the way down and fights on the way up. Combine energy conservation with the centripetal equation and the whole problem opens.

VERTICAL CIRCLE ON A STRING: the top is where it is decided



Just completes the loop when, at the top, gravity alone supplies the centripetal force:

v(top) = √(gr) and v(bottom) = √(5gr)

Tension at the bottom = 6mg
Tension at the top = 0 (in the critical case)

A RIGID ROD can push as well as pull, so it only needs v(top) = 0, giving v(bottom) = √(4gr).

The top is the critical point. If the string can just go slack there without the body falling away, it completes the loop.

★ CRITICAL SPEEDS ON A STRING

$$\text{At the top:}\quad T + mg = \frac{mv^2}{r} \Rightarrow v_{top}^{min} = \sqrt{gr}$$

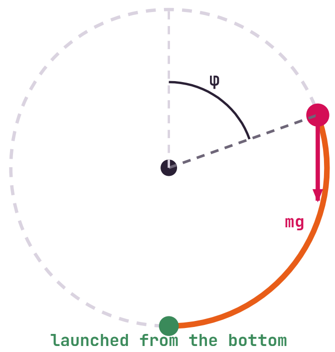
$$v_{bottom}^{min} = \sqrt{5gr}, \qquad T_{bottom} = 6mg$$

THINK IT THROUGH

A ROD IS NOT A STRING

A string can only pull, so it needs v_{top} = √(gr) to stay taut. A **rigid rod** can also push, so the body may creep over the top at nearly zero speed. That changes the requirement at the bottom from √(5gr) to √(4gr), and it is a favourite way to test whether you understood the reason rather than the formula.

TOO SLOW TO LOOP: the string goes slack part way up



Set the tension to zero at angle φ from the top:

$$mg\cos\varphi = mv^2/r \text{ with } v^2 = v(b)^2 - 2gr(1+\cos\varphi)$$

$$\cos\varphi = \left(v(b)^2 - 2gr \right) / \left(3gr \right)$$

Only happens when $\sqrt{2gr} < v(b) < \sqrt{5gr}$.
Below $\sqrt{2gr}$ it just swings back like a pendulum.

Launched too slowly to loop but too fast to swing back, the string goes slack somewhere above the horizontal and the body becomes a projectile.

▶ PLAY

VERTICAL CIRCLES FEEL LIKE MAGIC?

Set the launch speed and watch where the string lets go.

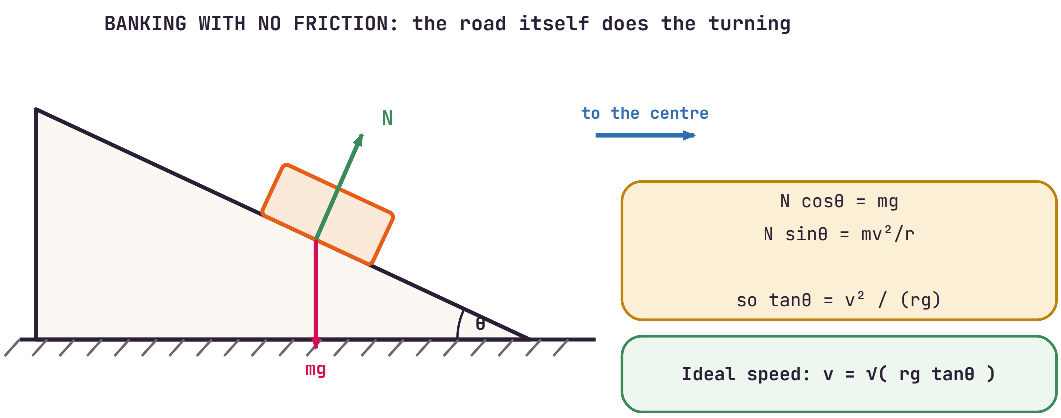
Slide the speed between $\sqrt{(2gr)}$ and $\sqrt{(5gr)}$ and see the slack point climb around the loop, then watch the body leave as a projectile. The three regimes become obvious. **logicbloom.co.in/download** · Free to start.

SCAN

6 Banking of Roads

IN NEET AND JEE

On a flat road only friction turns the car, which is unreliable in rain. Tilt the road and the normal reaction gains an inward component that does the job for free.



With no friction at all, the horizontal component of N is the entire centripetal force. That fixes exactly one speed for a given angle.

★ THE IDEAL SPEED

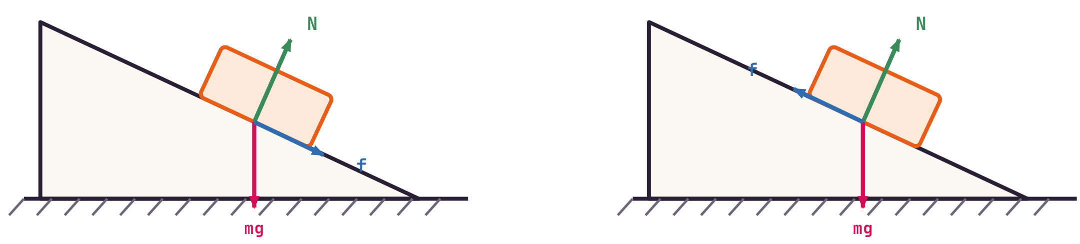
$$N\cos\theta = mg, \qquad N\sin\theta = \frac{mv^2}{r}$$

$$\tan\theta = \frac{v^2}{rg} \quad \Rightarrow \quad v_{ideal} = \sqrt{rg\tan\theta}$$

THINK IT THROUGH

WHERE THIS CONNECTS TO THE FRICTION SET

Real roads have friction, and now the same ceiling μN from the previous set reappears. Below the ideal speed the car tends to slide **down** the bank, so friction acts up the slope. Above it the car tends to ride **up**, so friction acts down the slope. Two directions, two limits.



The only difference between the two limiting cases is which way friction points. Get that right and the formulas write themselves.

★ THE TWO LIMITS WITH FRICTION

$$v_{max} = \sqrt{rg\frac{\tan\theta + \mu}{1 - \mu\tan\theta}}$$

$$v_{min} = \sqrt{rg\frac{\tan\theta - \mu}{1 + \mu\tan\theta}}$$

TRAP ALERT

⚠ TWO THINGS TO NOTICE ABOUT THOSE FORMULAS

If **μ ≥ tan θ** the expression for v_{min} goes negative, which means there is no minimum: the car can crawl round at any slow speed without sliding in. And on a **flat** road, θ = 0 reduces both to $v_{max} = \sqrt{\mu r g}$, the familiar result. Always sanity-check a banking formula by setting θ = 0.

▶ PLAY

BANKING FORMULAS HARD TO KEEP STRAIGHT?

Flip the speed and watch friction reverse.

Drive the car above and below the ideal speed and see the friction arrow turn around on the road surface. Once you have seen it flip, you never mis-sign the formula again. **logicbloom.co.in/download** · Free to start.

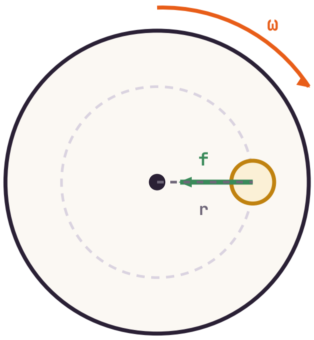
SCAN

7 Held In by Friction Alone

JEE ADVANCED MAINLY

Three classic setups where friction is the only thing preventing disaster. Each is a limiting-friction problem from the previous set, bent into a circle.

COIN ON A TURNTABLE: friction is the only thing holding it in



viewed from above

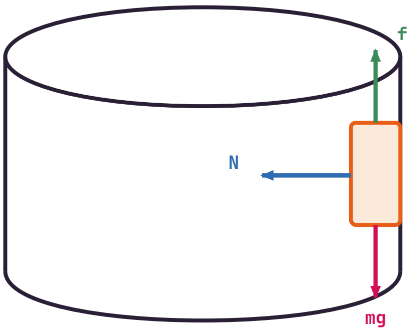
Friction must supply the whole centripetal force:

μmg ≥ mω²r so ω(max) = √(μg / r)

Speed the disc up and the OUTERMOST coin slips first, because the required force grows with r.

The coin needs friction to supply the whole of $m\omega^2 r$. Since that grows with r, the outermost coin always slips first.

THE ROTOR: the circular-motion twin of the block on a cart face



The wall pushes inward, and that N is what friction feeds on:

N = mω²r and f = μN must hold up mg

μmω²r ≥ mg so ω(min) = √(g / (μr))
The mass cancels, exactly as on the cart.

The rotor is the circular twin of the block pinned to an accelerating cart face: the wall supplies N, and friction on that N holds the weight.

★ **TWO FRICTION LIMITS**

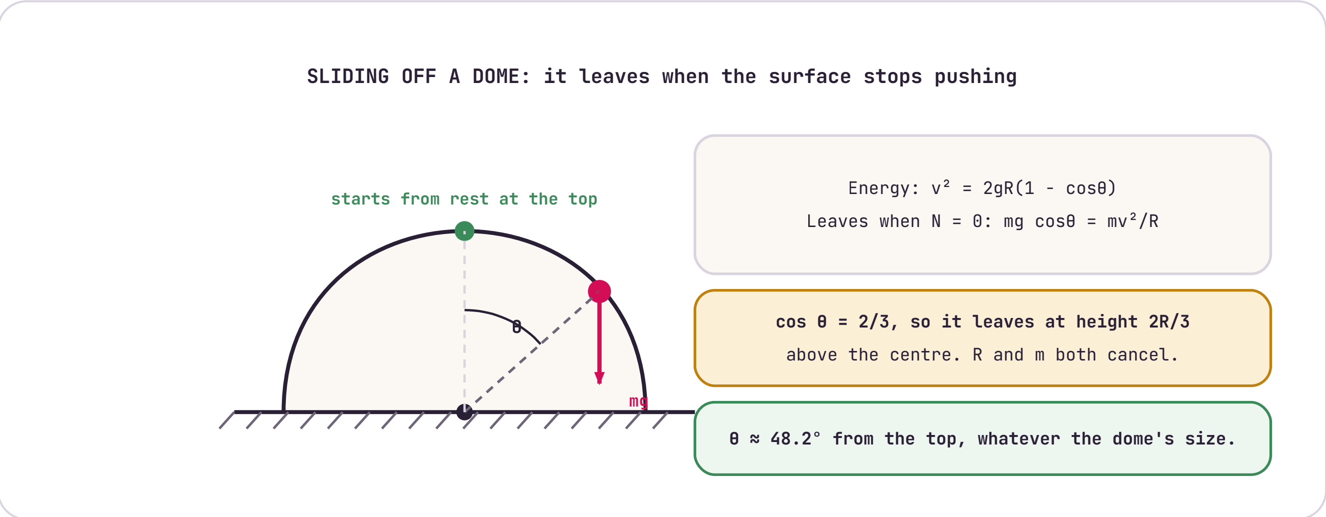
$$\text{Turntable:}\quad \mu mg \geq m\omega^2 r \Rightarrow \omega_{max} = \sqrt{\frac{\mu g}{r}}$$

$$\text{Rotor:}\quad \mu m\omega^2 r \geq mg \Rightarrow \omega_{min} = \sqrt{\frac{g}{\mu r}}$$

TRAP ALERT

⚠ THESE TWO LOOK ALIKE AND POINT OPPOSITE WAYS

On a turntable, spinning **faster** makes the coin fly off, so there is a maximum ω. In a rotor, spinning faster presses you harder into the wall and keeps you up, so there is a minimum ω. Read which surface the normal force acts on before reaching for a formula.



Sliding off a dome. The normal force falls to zero at a fixed angle that depends on nothing at all.

8 Important Questions

Fourteen questions, tagged by exam. Each needs at least two chained steps or a decision made correctly before any formula helps. Try each before reading the working.

Q1 Speed is changing too

NEET

A particle starts from rest on a circular track of radius 2 m and speeds up uniformly at 2 m/s^2. Find the magnitude of its total acceleration 2 s after starting.

CORE There are two perpendicular accelerations here: tangential a_t and centripetal $a_c = v^2/r$. Combine them as vectors.

WORKING

$$v = a_t t = 2(2) = 4 \text{ m/s} \Rightarrow a_c = \frac{4^2}{2} = 8 \text{ m/s}^2$$

$$|a| = \sqrt{a_t^2 + a_c^2} = \sqrt{2^2 + 8^2} = \mathbf{8.25} \text{ m/s}^2$$

TRAP Answering 8 m/s^2 means you forgot the tangential part; answering 2 means you forgot the circle. The total acceleration also does not point at the centre here, it leans 14° off the radius.

Q2 How sharply does a projectile turn

JEE MAIN

A projectile is launched at 20 m/s at 60° to the horizontal (g = 10). Find the radius of curvature of its path (a) at the highest point and (b) at the moment of launch.

CORE Use $r = v^2/a_\perp$. Only the component of g perpendicular to the velocity bends the path.

WORKING

$$\text{(a) top: } v = u\cos 60^\circ = 10,\ a_\perp = g \Rightarrow r = \frac{100}{10} = \mathbf{10} \text{ m}$$

$$\text{(b) launch: } v = 20,\ a_\perp = g\cos 60^\circ = 5 \Rightarrow r = \frac{400}{5} = \mathbf{80} \text{ m}$$

TRAP At launch a_{perp} is g cos θ, not g. Using the full g gives 40 m. The path is flattest, meaning the radius is largest, where the speed is greatest and the bending weakest.

Q3 Curvature straight from the vectors

JEE ADVANCED

At a certain instant a particle has velocity $\vec{v} = 3\hat{i} + 4\hat{j}$ m/s and acceleration $\vec{a} = 2\hat{i} + \hat{j}$ m/s². Find the radius of curvature of its path at that instant.

CORE Split a into parts along and across v. Only the perpendicular part curves the path, and $a_\perp = |\vec{v} \times \vec{a}|/|\vec{v}|$.

WORKING

$$|\vec{v}| = 5, \quad |\vec{v} \times \vec{a}| = |3(1) - 4(2)| = 5$$

$$a_\perp = \frac{5}{5} = 1 \text{ m/s}^2 \Rightarrow r = \frac{v^2}{a_\perp} = \frac{25}{1} = \mathbf{25} \text{ m}$$

TRAP Using |a| = √5 instead of a_{perp} gives 11.2 m and is wrong. The component of a along v is changing the speed, not the direction, so it must be discarded before dividing.

Q4 The cone swept by a string

NEET

A 0.5 kg bob on a 1 m string moves in a horizontal circle with the string at 60° to the vertical (g = 10). Find the tension and the time for one revolution.

CORE Resolve the tension: vertical component balances the weight, horizontal component turns the bob.

WORKING

$$T\cos 60^\circ = mg \Rightarrow T = \frac{0.5(10)}{0.5} = \mathbf{10} \text{ N}$$

$$\text{Period} = 2\pi\sqrt{\frac{L\cos\theta}{g}} = 2\pi\sqrt{\frac{1(0.5)}{10}} = \mathbf{1.41} \text{ s}$$

TRAP The tension is 10 N while the weight is only 5 N. Anyone who writes T = mg has resolved nothing. Note also that the period does not contain the mass at all.

Q5 Just barely round the loop

JEE ADVANCED

A 0.5 kg ball on a 1 m string is swung in a vertical circle (g = 10). Find the minimum speed at the lowest point for it to complete the circle, and the string tension there at that speed.

CORE The critical condition is at the **top**, where gravity alone must supply mv^2/r. Then use energy conservation to bring it down to the bottom.

WORKING

$$\text{top: } mg = \frac{mv_{top}^2}{r} \Rightarrow v_{top} = \sqrt{gr} = \sqrt{10}$$

$$\text{energy: } v_{bot}^2 = v_{top}^2 + 4gr = 10 + 40 = 50 \Rightarrow v_{bot} = \mathbf{7.07}\text{ m/s}$$

$$\text{bottom: } T - mg = \frac{mv_{bot}^2}{r} \Rightarrow T = 0.5(10) + \frac{0.5(50)}{1} = \mathbf{30}\text{ N}$$

TRAP The tension at the bottom is 6mg, six times the weight. Applying the critical condition at the bottom instead of the top is the classic error, and it makes the problem unsolvable.

Q6 Where the string gives up

JEE ADVANCED

A ball on a 1.5 m string is launched from the lowest point with $v^2 = 3gr$ (g = 10). Show that it cannot complete the circle, and find the angle from the top at which the string goes slack.

CORE Slack means T = 0. Write the radial equation at angle φ from the top and combine it with energy conservation from the bottom.

WORKING

$$v_b = \sqrt{3gr} = 6.7\text{ m/s} < \sqrt{5gr} = 8.7\text{ m/s} \Rightarrow \text{cannot loop}$$

$$T = 0:\ mg\cos\varphi = \frac{mv^2}{r}, \qquad v^2 = v_b^2 - 2gr(1+\cos\varphi)$$

$$\cos\varphi = \frac{v_b^2 - 2gr}{3gr} = \frac{1}{3} \Rightarrow \varphi = \mathbf{70.5^\circ}\text{ from the top}$$

TRAP Above the horizontal the string can go slack; below it the string stays taut whatever the speed, because gravity then has a component away from the centre. Also note the ball becomes a projectile from that moment, not a pendulum.

Q7 The road that turns the car for you

NEET

A curve of radius 100 m is banked at 30° (g = 10). Find the speed at which a car can round it with no friction at all.

CORE With no friction, the horizontal component of the normal reaction is the only inward force.

WORKING

$$\tan\theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg\tan\theta}$$

$$v = \sqrt{100(10)\tan 30^\circ} = \sqrt{577.4} = \mathbf{24.0}\text{ m/s}$$

TRAP This is one exact speed, not a maximum. On a genuinely frictionless bank a car going slower would slide inward and one going faster would slide outward.

Q8 Both limits on a real road

JEE ADVANCED

The same 100 m curve banked at 30° now has μ = 0.4 between tyres and road (g = 10). Find the maximum and minimum safe speeds.

CORE Friction reverses between the two cases. At v_{max} the car tends to slide up the bank so friction acts down it; at v_{min} the reverse.

WORKING

$$v_{max} = \sqrt{rg\frac{\tan\theta + \mu}{1 - \mu\tan\theta}} = \sqrt{1000 \cdot \frac{0.577 + 0.4}{1 - 0.231}} = \mathbf{35.7}\text{ m/s}$$

$$v_{min} = \sqrt{rg\frac{\tan\theta - \mu}{1 + \mu\tan\theta}} = \sqrt{1000 \cdot \frac{0.577 - 0.4}{1.231}} = \mathbf{12.0}\text{ m/s}$$

TRAP The ideal 24.0 m/s from the previous question sits between the two, as it must. Had μ been 0.6, larger than tan 30° = 0.577, the v_{min} expression would turn negative, meaning the car could stop on the bank without sliding in.

Q9 Was the banking worth it

JEE MAIN

A flat road of radius 50 m has μ = 0.5 (g = 10). Find the maximum safe speed, and state what changes if the same road is banked.

CORE On a flat road friction is the entire centripetal force, so $\mu mg = mv^2/r$.

WORKING

$$v_{max} = \sqrt{\mu rg} = \sqrt{0.5(50)(10)} = \sqrt{250} = \mathbf{15.8}\text{ m/s}$$

$$\text{banking adds the } N\sin\theta \text{ term, so the same } \mu \text{ then allows a higher } v_{max}$$

TRAP This is the θ = 0 case of the banking formula, which is the quickest way to check you have remembered it correctly. If your general formula does not collapse to √(μrg) at θ = 0, it is wrong.

Q10 Which coin flies off first

JEE MAIN

Coins are placed on a turntable at 10 cm and 20 cm from the centre, with μ = 0.5 (g = 10). The turntable is slowly speeded up. Find the angular speed at which the first coin slips, and say which one it is.

CORE Friction supplies the whole centripetal force, so the condition is $\mu mg \geq m\omega^2 r$. Note the mass cancels but the radius does not.

WORKING

$$\omega_{max} = \sqrt{\frac{\mu g}{r}} \Rightarrow \text{smaller for LARGER } r$$

$$r = 0.2:\ \omega = \sqrt{\frac{0.5(10)}{0.2}} = \mathbf{5}\text{ rad/s, the outer coin slips first}$$

TRAP The outer coin needs more centripetal force but has exactly the same friction available, so it always loses first. The mass cancelling means a heavy coin and a light coin at the same radius slip together.

Q11 Pinned to the wall of a spinning drum

JEE ADVANCED

In a rotor, a person stands against the inside wall of a cylindrical drum of radius 2.5 m. The floor is then removed. With μ = 0.4 (g = 10), find the minimum angular speed for the person not to slide down.

CORE The wall supplies N inward, and that N is what friction feeds on. Set the friction ceiling equal to the weight.

WORKING

$$N = m\omega^2 r, \qquad f_{max} = \mu N = \mu m\omega^2 r$$

$$\mu m\omega^2 r \geq mg \Rightarrow \omega_{min} = \sqrt{\frac{g}{\mu r}} = \sqrt{\frac{10}{1}} = \mathbf{3.16}\ \text{rad/s}$$

TRAP This is a **minimum**, the opposite of the turntable, because here spinning faster helps. It is also the same problem as the block on the accelerating cart face from the friction set, with $\omega^2 r$ in place of a.

Q12 A rod instead of a string

JEE ADVANCED

The 1 m string in the vertical-circle problem is replaced by a light rigid rod (g = 10). Find the new minimum speed at the lowest point, and explain the difference.

CORE A rod can push as well as pull, so it can support the body from below at the top. The critical speed at the top becomes zero.

WORKING

$$\text{string: } v_{top} = \sqrt{gr} \Rightarrow v_{bot} = \sqrt{5gr} = 7.07\ \text{m/s}$$

$$\text{rod: } v_{top} = 0 \Rightarrow v_{bot}^2 = 0 + 4gr = 40 \Rightarrow v_{bot} = \mathbf{6.32}\ \text{m/s}$$

TRAP Applying $\sqrt{(5gr)}$ to a rod is the single most common error in this topic. The 5 comes from the string's inability to push, so it has no business appearing in a rod problem.

Q13 Assertion and reason on centrifugal force

JEE ADVANCED

Assertion: The centrifugal force is the Newton's third law reaction to the centripetal force. **Reason:** Action and reaction are always equal and opposite. Judge both statements.

CORE Check the defining requirement of a third-law pair: the two forces must act on **different bodies**.

WORKING

$$\text{centripetal force acts on the body; its reaction acts on the string or road}$$

$$\text{centrifugal force acts on the SAME body, only in a rotating frame}$$

$$\text{Assertion false, Reason true but unrelated}$$

TRAP Centrifugal force is a pseudo-force, so it has no third-law partner at all. The reason is a correct statement of the third law, which is exactly what makes this question effective: a true reason is attached to a false assertion.

Q14 Sliding off a dome

JEE ADVANCED

A small block slides from rest at the top of a smooth hemispherical dome of radius 3 m (g = 10). Find the angle from the top at which it leaves the surface, and its speed at that moment.

CORE Combine energy conservation with the radial equation, and use the fact that leaving the surface means the normal reaction has fallen to zero.

WORKING

$$\text{energy: } v^2 = 2gR(1-\cos\theta)$$

$$\text{radial, with } N=0:\ mg\cos\theta = \frac{mv^2}{R} \Rightarrow gR\cos\theta = 2gR(1-\cos\theta)$$

$$3\cos\theta = 2 \Rightarrow \theta = \mathbf{48.2^\circ}, \quad v = \sqrt{2(10)(3)(1/3)} = \mathbf{4.47}\ \text{m/s}$$

TRAP Both R and m vanish from the angle, so every smooth dome sheds its load at the same 48.2°. Setting N = mg cos θ equal to zero rather than setting N itself to zero is the usual slip.

★ Circular Motion · Fact Sheet

Every result for revision day. Print this page alone.

THE DICTIONARY

$\boxed{v = \omega r,\ a_t = \alpha r}$

$a_c = v^2/r = \omega^2 r$

$\omega = 2\pi/T = 2\pi f$.

TOTAL ACCELERATION

$\boxed{|a| = \sqrt{(a_c^{\,2} + a_t^{\,2})}}$

Uniform circular motion is the special case $a_t = 0$.

RADIUS OF CURVATURE

$\boxed{r = v^2/a_{perp}}$

Projectile top: $r = v^2/g$

At launch: $r = u^2/(g\cos\theta)$.

CENTRIPETAL FORCE

It is a JOB, not a new force.

Tension, friction, gravity or N does it. Never add it separately.

CONICAL PENDULUM

$\boxed{T\cos\theta = mg,\ T\sin\theta = mv^2/r}$

$\tan\theta = v^2/rg$

Period $= 2\pi\sqrt{(L\cos\theta/g)}$.

VERTICAL CIRCLE, STRING

$\boxed{v_{top} = \sqrt{(gr)}}$

$v_{bottom} = \sqrt{(5gr)}$

$T_{bottom} = 6mg$.

VERTICAL CIRCLE, ROD

A rod can push, so $v_{top} = 0$

$v_{bottom} = \sqrt{(4gr)}$

Never use 5gr for a rod.

STRING GOES SLACK

$\boxed{\cos\phi = (v_b^{\,2} - 2gr)/(3gr)}$

Only for $\sqrt{(2gr)} < v_b < \sqrt{(5gr)}$ and always above the horizontal.

BANKING, NO FRICTION

$\boxed{\tan\theta = v^2/rg}$

$v_{ideal} = \sqrt{(rg\tan\theta)}$

One exact speed, not a range.

BANKING WITH FRICTION

$\boxed{v_{max} = \sqrt{[rg(\tan\theta+\mu)/(1-\mu\tan\theta)]}}$

$v_{min} = \sqrt{[rg(\tan\theta-\mu)/(1+\mu\tan\theta)]}$

At $\theta = 0$ both give $\sqrt{(\mu rg)}$.

TURNTABLE

$\boxed{\mu mg \geq m\omega^2 r}$

$\omega_{max} = \sqrt{(\mu g/r)}$

The outermost coin slips first.

ROTOR AND DOME

$\boxed{\text{Rotor: } \omega_{min} = \sqrt{(g/\mu r)}}$

Dome: leaves at $\cos\theta = 2/3$ that is 48.2°, whatever R.

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