



LOGIC BLOOM · MATHEMATICS · CLASS 11

# Binomial Theorem

Expansion · general term · coefficient identities

**JEE Main and JEE Advanced**

*Every coefficient in a binomial expansion is a count: the number of ways to choose. Hold on to that one idea and the whole chapter, from Pascal's triangle to the hardest identity, turns into careful counting.*

11 Sections

10 Diagrams

Advanced Tier

Worked Problems

Fact Sheet

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understand the NCERT.*

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# How to Read This Set

Sections 1 to 8 cover the full syllabus for **JEE Main**. Sections 9 and 10 are the **JEE Advanced** layer. There the answer is never one formula away. You have to build the route yourself.

## THE WHOLE CHAPTER IN ONE LINE

### A binomial coefficient counts the ways to choose.

Every result here follows from that. The expansion, Pascal's rule and the coefficient identities are all careful counting.



THINK IT  
THROUGH

## WHAT THIS CHAPTER IS WORTH

- It appears in **almost every JEE Main paper**, usually as one or two questions.
- The questions are short, so a clear method scores quickly. A vague one burns minutes.
- Its identities come back later in probability and in sequences and series.



TRAP ALERT

## THE HABIT THAT FIXES THIS CHAPTER

- **Write the general term first, every time.** Then read the answer off it.
- Almost every mistake comes from guessing a term number instead of solving for  $r$ .
- Remember that  $r$  is the power of the **second** term, and the term is  $T_{r+1}$ .

## Where the Coefficients Come From

many ways lead

Picking  $b$  from exactly two of four brackets can be done in six ways. That count is the coefficient.

Picking  $b$  from exactly two of four brackets can be done in six ways. That count is the coefficient.

THE BINOMIAL THEOREM, FOR A POSITIVE WHOLE NUMBER  $n$

$$(a+b)^n = \sum_{r=0}^n {}^nC_r a^{n-r} b^r$$

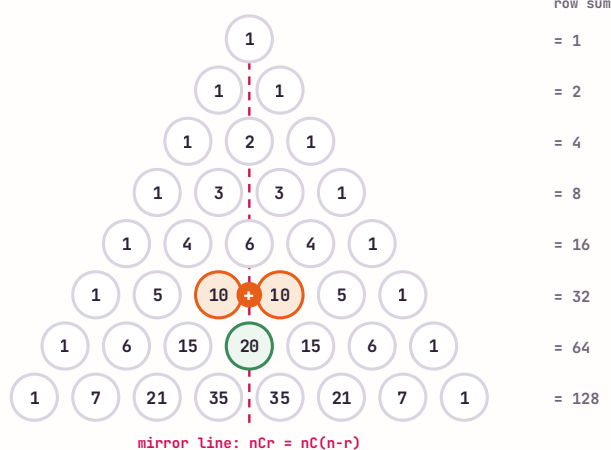
where  ${}^nC_r = \frac{n!}{r!(n-r)!}$

$$(a+b)^n = \sum_{r=0}^n {}^nC_r a^{n-r} b^r$$

where  ${}^nC_r = \frac{n!}{r!(n-r)!}$

The same coefficients can be built without any factorials. Each number is the sum of the two just above it.

PASCAL'S TRIANGLE: EACH NUMBER IS THE SUM OF THE TWO ABOVE IT



PASCAL'S RULE

$$nC_r + nC_{r+1} = (n+1)C_{r+1}$$

10 + 10 = 20 above

SYMMETRY

each row reads the same both ways

ROW  $n$

gives the coefficients of  $(a+b)^n$

row  $n$  adds up to  $2^n$

Row  $n$  gives the coefficients of  $(a+b)$  to the power  $n$ . The two 10s add to make the 20 below.

PASCAL'S RULE

$$nC_r + nC_{r+1} = {}^{n+1}C_{r+1}$$

Choosing  $r+1$  from  $n+1$  things: either the last thing is chosen, or it is not. Those two cases give the two terms.

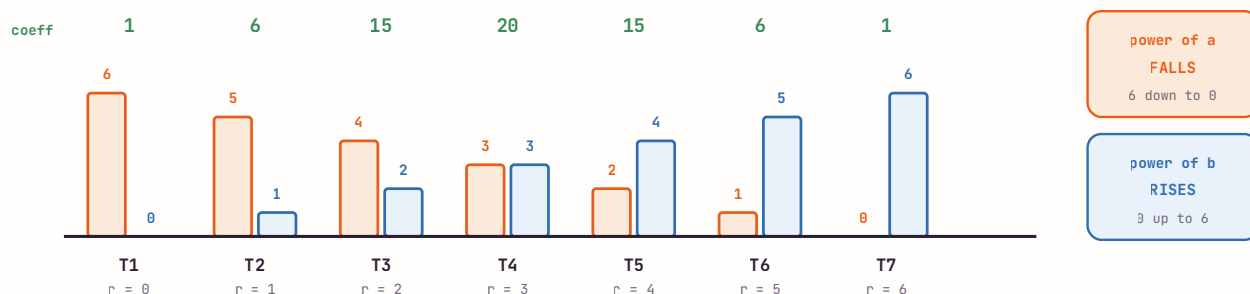
| PROPERTY            | STATEMENT                                 |
|---------------------|-------------------------------------------|
| Symmetry            | $nC_r = nC_{n-r}$                         |
| Edges               | $nC_0 = nC_n = 1$                         |
| Next to the edge    | $nC_1 = nC_{n-1} = n$                     |
| Pull out a factor   | $nC_r = \frac{n}{r} {}^{n-1}C_{r-1}$      |
| Ratio of neighbours | $\frac{nC_r}{nC_{r-1}} = \frac{n-r+1}{r}$ |



THINK IT  
THROUGH

USE THE RATIO OF NEIGHBOURS

- The last row of that table does most of the work in sections 4 and 5.
- It tells you whether the coefficients are still rising or have started to fall.
- It needs no factorials at all, so it is fast in an exam.

EVERY TERM OF  $(a+b)^6$ , SIDE BY SIDE

power of a  
**FALLS**  
6 down to 0

power of b  
**RISES**  
0 up to 6

$r$  is the power of the **SECOND** term, and the term is  $T(r+1)$ , not  $T(r)$ .  
The two powers always add to  $n$ , and there are  $n + 1$  terms in all.

Seven terms of  $(a+b)$  to the sixth. The power of the second term is always  $r$ , one less than the term number.

## THE GENERAL TERM

$$T_{r+1} = {}^nC_r a^{n-r} b^r$$

The term that is  $r + 1$  places from the start. The term  $r + 1$  places from the **end** is  $T_{n-r+1}$  from the start.

## WORKED EXAMPLE 1

**Find the coefficient of  $x^3$  in  $(2x+3)^7$ .**

General term:  $T_{r+1} = {}^7C_r (2x)^{7-r} 3^r$ .

The power of  $x$  is  $7 - r$ . Set  $7 - r = 3$ , so  $r = 4$ .

Coefficient  $= {}^7C_4 \cdot 2^3 \cdot 3^4 = 35 \times 8 \times 81$ .

**Answer: 22680**

## WORKED EXAMPLE 2

**Find the term independent of  $x$  in  $(x^2 - \frac{1}{x})^9$ .**

General term:  $T_{r+1} = {}^9C_r (x^2)^{9-r} \left(-\frac{1}{x}\right)^r = {}^9C_r (-1)^r x^{18-3r}$ .

Independent of  $x$  means the power is zero. So  $18 - 3r = 0$ , giving  $r = 6$ .

The term is  $T_7 = {}^9C_6 (-1)^6 = 84$ .

**Answer: 84, the seventh term**

### WORKED EXAMPLE 3

How many rational terms are there in  $(\sqrt{2} + \sqrt[3]{3})^{100}$ ?

General term:  $T_{r+1} = {}^{100}C_r 2^{(100-r)/2} 3^{r/3}$ .

Both powers must be whole numbers. So  $100 - r$  is even, which means  $r$  is even. And  $r$  is a multiple of 3.

So  $r$  is a multiple of 6: 0, 6, 12, and so on up to 96.

**Answer: 17 rational terms**

### WORKED EXAMPLE 4

Find the 4th term from the end of  $(2x - \frac{1}{x})^{10}$ .

There are 11 terms. Counting 4 from the end lands on term  $11 - 4 + 1 = 8$  from the start.

So use  $r = 7$ :  $T_8 = {}^{10}C_7 (2x)^3 \left(-\frac{1}{x}\right)^7 = 120 \times 8 \times (-1)x^{3-7}$ .

Quick check: the  $k$ -th term from the end is the  $(n - k + 2)$ -th from the start.

**Answer:  $-\frac{960}{x^4}$**



TRAP ALERT

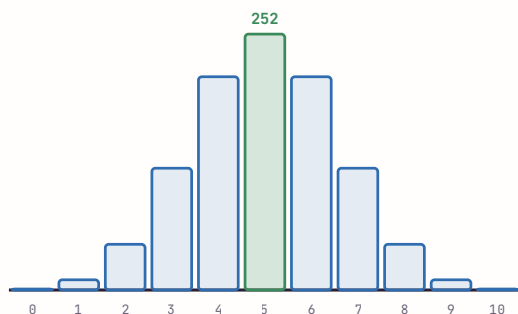
### THREE SLIPS WORTH AVOIDING

- **Calling it  $T_r$ .** The formula gives  $T_{r+1}$ . Mixing these up shifts every answer by one term.
- **Dropping the sign.** In  $(x - y)^n$  the second term is  $-y$ , so a factor  $(-1)^r$  appears.
- **Forgetting to raise the number as well as the  $x$ .** In  $(2x)^{7-r}$  the 2 is raised too.

## Middle Terms and the Greatest Coefficient

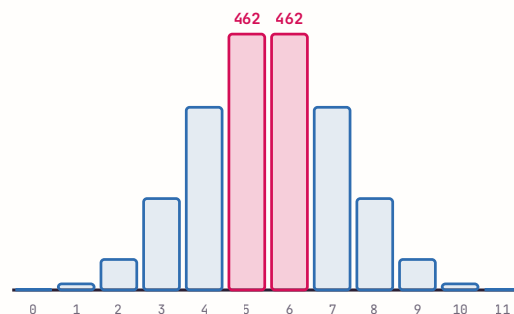
The binomial coefficients rise to a peak in the middle, then fall again. Where that peak sits depends only on whether  $n$  is even or odd.

THE GREATEST COEFFICIENT: ONE PEAK OR TWO



$n = 10$ , EVEN

one peak:  ${}^{10}C_5 = 252$ , the middle term



$n = 11$ , ODD

two equal peaks:  ${}^{11}C_5 = {}^{11}C_6 = 462$

$n$  even: ONE middle term  $T_{(n/2 + 1)}$ .  $n$  odd: TWO middle terms  $T_{((n+1)/2)}$  and  $T_{((n+3)/2)}$ .

*An even power has one tallest bar. An odd power has two equal tallest bars.*

| N IS | MIDDLE TERM(S)                       | GREATEST BINOMIAL COEFFICIENT       |
|------|--------------------------------------|-------------------------------------|
| even | one: $T_{n/2+1}$                     | ${}^nC_{n/2}$                       |
| odd  | two: $T_{(n+1)/2}$ and $T_{(n+3)/2}$ | ${}^nC_{(n-1)/2} = {}^nC_{(n+1)/2}$ |

### WORKED EXAMPLE

Find the middle term of  $(x + \frac{1}{x})^{10}$ .

$n = 10$  is even, so there is one middle term,  $T_6$ , with  $r = 5$ .

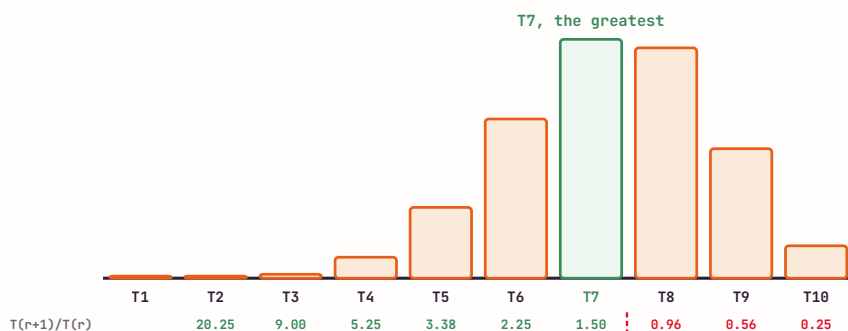
$$T_6 = {}^{10}C_5 x^5 \left(\frac{1}{x}\right)^5 = {}^{10}C_5.$$

**Answer: 252, and it is independent of  $x$**

# The Greatest Term

The greatest **term** is a different question from the greatest **coefficient**. The size of a term depends on the value of  $x$ . So the peak moves when  $x$  changes.

THE GREATEST TERM OF  $(2 + 3x)^9$  AT  $x = 3/2$



## THE RATIO TEST

keep going while  
 $T(r+1)/T(r)$  is above 1  
 stop the moment it drops below

## THE SHORTCUT

$m = (n+1)|y| / (1+|y|)$   
 here  $90/13 = 6.9$ , so T7

Same expansion, THREE different answers. Read which one the question wants.

Greatest BINOMIAL coefficient  ${}^9C_r$ : T5 and T6. Greatest NUMERICAL coefficient: T6 and T7, a tie.

Greatest TERM at  $x = 3/2$ : T7 alone. Only the last one changes if  $x$  changes.

Keep moving right while the ratio of neighbouring terms stays above 1.

## THE METHOD

Compare neighbours:  $\frac{T_{r+1}}{T_r} = \frac{n-r+1}{r} \cdot \left| \frac{b}{a} \right|$

Terms keep rising while this ratio is above 1. The last one before it drops is the greatest.

Shortcut: work out  $m = \frac{(n+1)|y|}{1+|y|}$  with  $y = \frac{b}{a}$ .

If  $m$  is not a whole number,  $T_{[m]+1}$  is greatest. If it is,  $T_m$  and  $T_{m+1}$  tie.

FOR  $(2 + 3x)^9$  AT  $x = 3/2$ , THE GREATEST ...

IS AT

DEPENDS ON  $x$ ?

binomial coefficient  ${}^9C_r$

T5 and T6

no

numerical coefficient, the full number in front of  $x$

T6 and T7, a tie

no

term, with  $x$  put in

T7 alone

yes

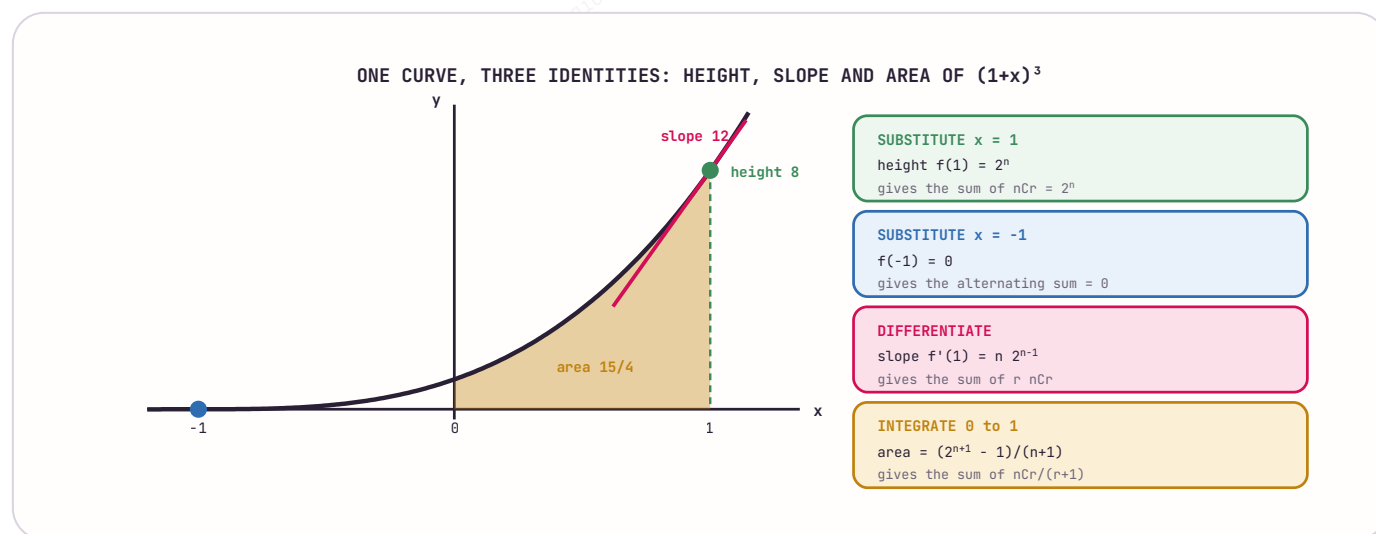


TRAP ALERT

## READ WHICH GREATEST THE QUESTION WANTS

- **Binomial coefficient** means  ${}^nC_r$  alone.
- **Numerical coefficient** includes the constants from inside the bracket.
- **Term** includes  $x$  as well. It is the only one of the three that changes with  $x$ .
- One expansion can give three different answers, as the table shows.

Start from one expansion,  $(1+x)^n = \sum^n C_r x^r$ . Every identity comes from doing one of four things to both sides.



Height at  $x = 1$ , the value at  $x = -1$ , the slope at  $x = 1$ , and the area from 0 to 1.

| IDENTITY                       | VALUE                     | HOW YOU GET IT                                       |
|--------------------------------|---------------------------|------------------------------------------------------|
| $\sum^n C_r$                   | $2^n$                     | put $x = 1$                                          |
| $\sum (-1)^r {}^n C_r$         | 0                         | put $x = -1$                                         |
| sum of even-place coefficients | $2^{n-1}$                 | add the two results above                            |
| $\sum r^n C_r$                 | $n 2^{n-1}$               | differentiate, then put $x = 1$                      |
| $\sum r^2 {}^n C_r$            | $n(n+1) 2^{n-2}$          | differentiate, multiply by $x$ , differentiate again |
| $\sum \frac{{}^n C_r}{r+1}$    | $\frac{2^{n+1} - 1}{n+1}$ | integrate from 0 to 1                                |
| $\sum ({}^n C_r)^2$            | ${}^{2n} C_n$             | coefficient of $x^n$ in $(1+x)^n (x+1)^n$            |
| $\sum {}^n C_r {}^n C_{r+1}$   | ${}^{2n} C_{n-1}$         | same product, a different power of $x$               |
| $\sum r ({}^n C_r)^2$          | $n {}^{2n-1} C_{n-1}$     | pull out $n$ , then the product again                |

| IDENTITY                       | VALUE          | HOW YOU GET IT                                 |
|--------------------------------|----------------|------------------------------------------------|
| $\sum_k {}^m C_k {}^n C_{r-k}$ | ${}^{m+n} C_r$ | Vandermonde: compare $x^r$ in $(1+x)^m(1+x)^n$ |



THINK IT  
THROUGH

#### WHY THE PRODUCT TRICK WORKS FOR THE SUM OF SQUARES

- Write  $(1+x)^n$  twice, but the second one backwards, as  $(x+1)^n$ .
- To get  $x^n$  in the product, take  $x^r$  from the first and  $x^{n-r}$  from the second.
- Those coefficients are  ${}^n C_r$  and  ${}^n C_{n-r}$ , which is  ${}^n C_r$  again. So each pair gives a square.
- The product is also  $(1+x)^{2n}$ , whose  $x^n$  coefficient is  ${}^{2n} C_n$ .



▶ PLAY

#### IDENTITIES FEELING LIKE A LIST?

**Watch each one appear on the curve.**

Slide x, draw the tangent, shade the area, and see each identity come out as a height, a slope or an area. [logicbloom.co.in/download](https://logicbloom.co.in/download) • Free to start.

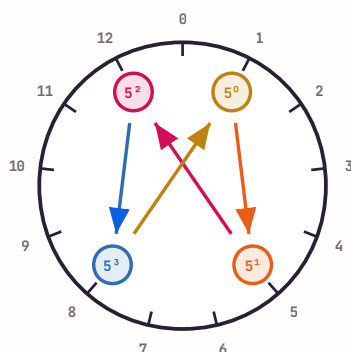


SCAN

# Remainders and Divisibility

To find a remainder, rewrite the base as **a multiple of the divisor, plus or minus 1**. Then expand. Every term except the last carries the divisor as a factor.

POWERS OF 5 ON A 13-HOUR CLOCK: THEY REPEAT EVERY 4



## THE BINOMIAL MOVE

$25 = 26 - 1$ , and 26 is a multiple of 13. So  $5^2$  sits at -1, which is 12.  
Then  $5^{98} = (26 - 1)^{49}$ . Every term except the last has a factor 26.

## SO

$5^{98}$  leaves  $(-1)^{49} = -1$ . Then  $5^{99}$  leaves -5, which is 8.  
The clock agrees:  $99 = 4(24) + 3$ , and  $5^3$  lands on 8.

Always rewrite the base as (a multiple of the divisor) plus or minus 1.

Powers of 5 cycle through four positions on a 13-hour clock. The binomial move gets there directly.

## WORKED EXAMPLE 1

**Find the remainder when  $5^{99}$  is divided by 13.**

$5^2 = 25 = 26 - 1$ , and 26 is a multiple of 13.

$5^{99} = 5 \cdot (26 - 1)^{49}$ . Every term of  $(26 - 1)^{49}$  has a factor 26, except the last,  $(-1)^{49} = -1$ .

So  $5^{99}$  leaves  $5 \times (-1) = -5$ . Add 13 to make it positive.

**Answer: 8**

## WORKED EXAMPLE 2

**Find the last two digits of  $7^{100}$ .**

$7^4 = 2401 = 2400 + 1$ , and 2400 is a multiple of 100.

$7^{100} = (1 + 2400)^{25}$ . Every term after the first has a factor 2400.

**Answer: 01**

### WORKED EXAMPLE 3

Show that  $3^{2n+2} - 8n - 9$  is divisible by 64 for every positive whole number  $n$ .

$$3^{2n+2} = 9^{n+1} = (1+8)^{n+1}.$$

Expand:  $1 + (n+1) \cdot 8 + {}^{n+1}C_2 \cdot 8^2 + {}^{n+1}C_3 \cdot 8^3 + \dots$

The first two terms give  $8n + 9$ , which cancels exactly. Every term left has  $8^2 = 64$  in it.

**Proved: the whole expression is a multiple of 64**



TRAP ALERT

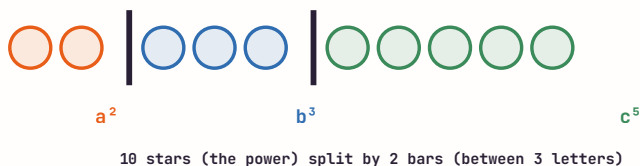
#### A NEGATIVE REMAINDER IS NOT THE ANSWER

- The method often ends at a value such as -5. **A remainder must lie between 0 and the divisor.**
- Add the divisor once:  $-5 + 13 = 8$ .

# The Multinomial Expansion

With three or more terms in the bracket, the same choosing idea still works. Now you choose which letter to take from each bracket, out of three or more options.

## COUNTING TERMS IN $(a + b + c)^{10}$ : STARS AND BARS



### HOW MANY TERMS

12 slots in a row,  
choose which 2 are bars

$${}^{12}C_2 = 66 \text{ terms}$$

### NUMBER OF TERMS

$(x_1 + x_2 + \dots + x_k)^n$   
has  $(n+k-1) C (k-1)$  terms

### ONE COEFFICIENT

$$a^2b^3c^5 : \frac{10!}{(2! 3! 5!)} = 2520$$

Ten stars for the power, two bars to split them among three letters.

## THE TWO RESULTS

Coefficient of  $a^p b^q c^s$  in  $(a + b + c)^n$  with  $p + q + s = n$ :  $\frac{n!}{p! q! s!}$

Number of terms in  $(x_1 + x_2 + \dots + x_k)^n$ :  ${}^{n+k-1}C_{k-1}$

## WORKED EXAMPLE

**Find the coefficient of  $x^4$  in  $(1 + x + x^2)^5$ .**

Take 1 from  $p$  brackets,  $x$  from  $q$  brackets and  $x^2$  from  $s$  brackets. Then  $p + q + s = 5$ , and the power is  $q + 2s = 4$ .

The options are  $(s, q, p) = (0, 4, 1), (1, 2, 2)$  and  $(2, 0, 3)$ .

They give  $\frac{5!}{1! 4! 0!} + \frac{5!}{2! 2! 1!} + \frac{5!}{3! 0! 2!} = 5 + 30 + 10$ .

**Answer: 45**



THINK IT  
THROUGH

## COUNTING TERMS WHEN POWERS CAN MERGE

- The formula counts terms in separate letters, such as  $a, b$  and  $c$ .
- In  $(1 + x + x^2)^5$  the letters are all powers of  $x$ , so many terms merge.
- The powers run from 0 to 10, so there are only **11** distinct terms, not 21.

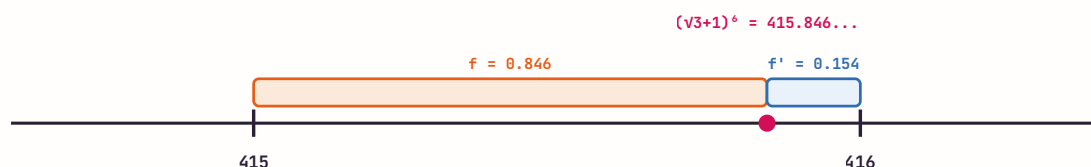
# The JEE Advanced layer

Everything so far is complete for JEE Main. From here each problem needs a route you construct yourself. No single formula reaches the answer.

## 9 Integral and Fractional Parts

A number like  $(\sqrt{3} + 1)^6$  is irrational, yet questions ask for its whole-number part. The trick is to pair it with its **conjugate**,  $(\sqrt{3} - 1)^6$ , which is small.

$(\sqrt{3} + 1)^6$  AND ITS PARTNER: WHY THE FRACTIONS ADD TO EXACTLY 1



### THE PAIR

$$(\sqrt{3}+1)^6 = I + f$$

$$(\sqrt{3}-1)^6 = f', \text{ a tiny number below } 1$$

Add them: the  $\sqrt{3}$  terms cancel. Sum = 416.

### THE ARGUMENT

$I + f + f'$  is a whole number,  
so  $f + f'$  must be too. Both lie in  $(0,1)$ ,

$$\text{so } f + f' = 1 \text{ and } I = 416 - 1 = 415.$$

The two fractional parts must add to exactly 1, because the total is a whole number.



▶ PLAY

### CONJUGATES STILL FEEL LIKE A TRICK?

**Push the power up and watch the two fractions keep adding to 1.**

Raise  $n$  one step at a time. The big number grows, the small partner shrinks, and  $f + f'$  stays fixed at exactly 1. [logicbloom.co.in/download](https://logicbloom.co.in/download) · Free to start.



SCAN

### THE GENERAL ARGUMENT

Let  $(\sqrt{a} + b)^n = I + f$  with  $I$  a whole number and  $0 < f < 1$ .

Let  $f' = (\sqrt{a} - b)^n$ , chosen so that  $0 < f' < 1$ .

Adding the two expansions cancels every odd power of  $\sqrt{a}$ . So  $I + f + f'$  is a whole number.

So  $f + f'$  is a whole number between 0 and 2. It must equal **1**.



THINK IT  
THROUGH

### WHEN THE PARTNER IS SUBTRACTED INSTEAD

- If  $f'$  is small but can be negative, use the difference  $I + f - f'$  instead.
- Then the **even** powers cancel, and  $f - f'$  is a whole number in  $(-1, 1)$ , so it is 0.
- So  $f = f'$ . Pick the sum or the difference by which one cancels the irrational part.

Each problem uses a different technique. None of them can be solved by recalling a formula.

### PROBLEM 1 • INTEGRATION

Find  $\sum_{r=0}^n \frac{(-1)^r {}^n C_r}{r+1}$ .

**What it demands:** turning the coefficient sum into an integral, and choosing the limits 0 and 1

Start from  $(1-x)^n = \sum (-1)^r {}^n C_r x^r$ .

Integrate both sides from 0 to 1. The right side becomes the sum we want, since  $\int_0^1 x^r dx = \frac{1}{r+1}$ .

The left side is  $\int_0^1 (1-x)^n dx = \frac{1}{n+1}$ .

**Answer:**  $\frac{1}{n+1}$

### PROBLEM 2 • HOCKEY STICK TWICE

Find the coefficient of  $x^{50}$  in  $\sum_{k=0}^{1000} (k+1)x^k(1+x)^{1000-k}$ .

**What it demands:** rewriting a weight as a count, swapping the order of summation, and applying the same identity twice

The term  $x^k(1+x)^{1000-k}$  contributes  ${}^{1000-k}C_{50-k}$  to  $x^{50}$ . So we need  $\sum_{k=0}^{50} (k+1) {}^{1000-k}C_{50-k}$ .

Write the weight  $k+1$  as a count: it is the number of  $j$  from 0 to  $k$ . Swap the order of summing.

For fixed  $j$ ,  $\sum_{k=j}^{50} {}^{1000-k}C_{50-k} = {}^{1001-j}C_{50-j}$ , by the hockey stick identity.

Then  $\sum_{j=0}^{50} {}^{1001-j}C_{50-j} = {}^{1002}C_{50}$ , by the hockey stick again.

**Answer:**  ${}^{1002}C_{50}$

**PROBLEM 3 • TRUNCATION**

**Find the last three digits of  $17^{256}$ .**

*What it demands:* choosing a base close to a round number, then deciding which terms can still matter

$$17^2 = 289 = 290 - 1. \text{ So } 17^{256} = (1 - 290)^{128}.$$

Work modulo 1000. Since  $290^3$  ends in 000, only three terms survive.

$$1 - 128(290) + {}^{128}C_2(290)^2 = 1 - 37120 + 8128 \times 84100.$$

Modulo 1000 these are 1, 880 and 800. Their sum is 1681.

**Answer: 681**

**PROBLEM 4 • PARITY**

**Show that the integral part of  $(5 + 2\sqrt{6})^n$  is odd for every positive whole number  $n$ .**

*What it demands:* a counterintuitive claim about an irrational number, settled by the conjugate and a parity argument

Let  $(5 + 2\sqrt{6})^n = I + f$ . Let  $f' = (5 - 2\sqrt{6})^n$ . Since  $5 - 2\sqrt{6} \approx 0.1$ , we have  $0 < f' < 1$ .

Adding cancels the odd powers of  $\sqrt{6}$ :  $I + f + f' = 2[5^n + {}^nC_2 5^{n-2}(24) + \dots]$ , an even number.

So  $f + f' = 1$ , and  $I = \text{even} - 1$ .

**Proved: I is always odd**

**PROBLEM 5 • OPTIMISATION**

**For which  $x > 0$  is  $T_6$  the greatest term of  $(1 + x)^{10}$ ?**

*What it demands:* turning 'greatest' into two simultaneous inequalities, then handling the equal cases

$T_6$  is greatest when  $T_6 \geq T_5$  and  $T_6 \geq T_7$ .

$$\frac{T_6}{T_5} = \frac{{}^{10}C_5}{{}^{10}C_4} x = \frac{6}{5}x \geq 1 \text{ gives } x \geq \frac{5}{6}.$$

$$\frac{T_7}{T_6} = \frac{{}^{10}C_6}{{}^{10}C_5} x = \frac{5}{6}x \leq 1 \text{ gives } x \leq \frac{6}{5}.$$

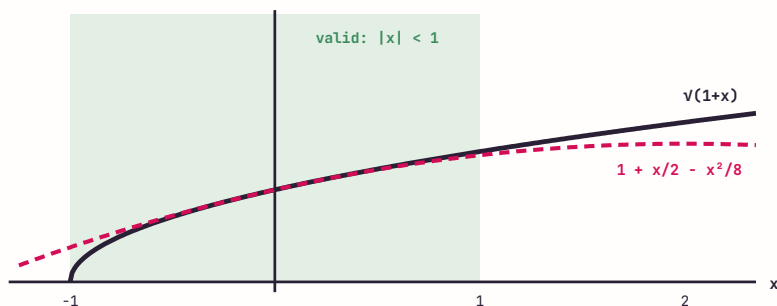
At the two ends there is a tie, with  $T_5$  or  $T_7$ .

**Answer:  $\frac{5}{6} \leq x \leq \frac{6}{5}$ , strictly greatest inside the interval**

## Beyond the Syllabus: Any Index

As we read them, the current JEE Main and JEE Advanced syllabi cover only a **positive whole number** index. The expansion below still turns up in older question banks and other exams. It is kept short, so skip it if time is tight.

$\sqrt{1+x}$  AGAINST ITS FIRST THREE TERMS: GOOD ONLY WHILE  $|x| < 1$



### THE APPROXIMATION

$\sqrt{1.02}$  with  $x = 0.02$ :  
 $1 + 0.01 - 0.00005$   
 $= 1.00995$ , true  $1.009950$

### THE TWO CONDITIONS

first term must be 1  
 and  $|x|$  must be below 1

The series follows the curve closely near 0, then parts company once  $x$  moves past 1.

FOR ANY REAL  $n$ , VALID ONLY WHEN  $|x| < 1$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$$

| EXPANSION    | COEFFICIENT OF         |
|--------------|------------------------|
| $(1-x)^{-1}$ | 1                      |
| $(1-x)^{-2}$ | $r+1$                  |
| $(1-x)^{-3}$ | $\frac{(r+1)(r+2)}{2}$ |
| $(1-x)^{-n}$ | $n+r-1C_r$             |



### TWO CONDITIONS, BOTH REQUIRED

- **The first term must be 1.** For  $(2+x)^n$ , first write it as  $2^n(1+\frac{x}{2})^n$ .
- **Then the variable part must lie between -1 and 1.** Outside that range the series does not settle to any value.

# ★ Binomial Theorem • Fact Sheet

Every rule for revision day. Print this page alone.

## THE THEOREM

$$(a + b)^n = \sum^n C_r a^{n-r} b^r$$

$n + 1$  terms.  
The powers add to  $n$ .

## GENERAL TERM

$$T_{r+1} = {}^nC_r a^{n-r} b^r$$

$r$  is the power of the SECOND term.  
From the end:  $T_{n-r+1}$ .

## PASCAL

$${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$$

Symmetric rows.  
Row  $n$  adds to  $2^n$ .

## MIDDLE TERMS

$$\mathbf{n \text{ even: one, } T_{n/2+1}}$$

$n$  odd: two, which tie for greatest coefficient.

## GREATEST TERM

$$m = \frac{(n+1)|y|}{1+|y|}$$

Not whole:  $T_{[m]+1}$ .  
Whole:  $T_m$  and  $T_{m+1}$  tie.

## THREE GREATESTS

**Binomial coeff, numerical coeff,**

and term can all differ.  
Only the term depends on  $x$ .

## SUMS

$$\sum^n C_r = 2^n$$

$\sum r^n C_r = n 2^{n-1}$   
even places = odd places.

## SQUARES

$$\sum ({}^nC_r)^2 = 2^n C_n$$

Vandermonde:  
 $\sum {}^m C_k {}^n C_{r-k} = {}^{m+n} C_r$ .

## CALCULUS

$$\sum \frac{{}^nC_r}{r+1} = \frac{2^{n+1}-1}{n+1}$$

Integrate 0 to 1.  
Differentiate for  $r$ -weights.

## REMAINDERS

**Base = multiple of divisor, plus or minus 1.**

Only the last term survives.  
Fix a negative remainder.

## MULTINOMIAL

$$\text{Coeff } \frac{n!}{p! q! s!}$$

Terms:  ${}^{n+k-1} C_{k-1}$   
Merging powers means fewer.

## INTEGRAL PART

**Pair with the conjugate.**

$f + f' = 1$  when  $f'$  is in  $(0,1)$ .  
 $(5 + 2\sqrt{6})^n$  has odd I.



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★ BINOMIAL STILL A LIST OF FORMULAS?

**Play it. Then practice it.**

Open the Binomial Theorem in the Logic Bloom app. Pick letters from brackets and watch the coefficient count itself, then drag x and watch the greatest term move while the coefficients stay put. Free to start.

SCAN → PLAY · READ · PRACTICE