PHYSICS FOUNDATIONS · JEE & NEET

# Basic Mathematics *for* Physics

*The Mathematical Toolkit — Calculus, Vectors, Graphs & Approximations*

Master the maths before the mechanics

**Logic Bloom**

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*Understand through games.*
*Score through practice.*

Physics is written in the language of mathematics, and NCERT Physics quietly *assumes* you already speak it. This chapter is that missing prerequisite: the calculus, trigonometry, vectors and graph-reading that every chapter from Kinematics to Electromagnetism leans on. The treatment is **JEE-heavy on calculus** (genuine differentiation, definite integration, and maxima–minima applied *inside* physics), with NEET-level essentials clearly covered too.

> **HOW TO USE THESE NOTES**
>
> Each topic gives the concept, a clear diagram, worked physics-flavoured examples, the **traps** that cost marks, and **Advanced** boxes for deeper JEE technique. Do not memorise formulae in isolation: every result here exists because a physics problem needs it. The one-page **formula sheet** at the end is your revision-day companion.

# Contents

M01

# Algebra & the Binomial Approximation

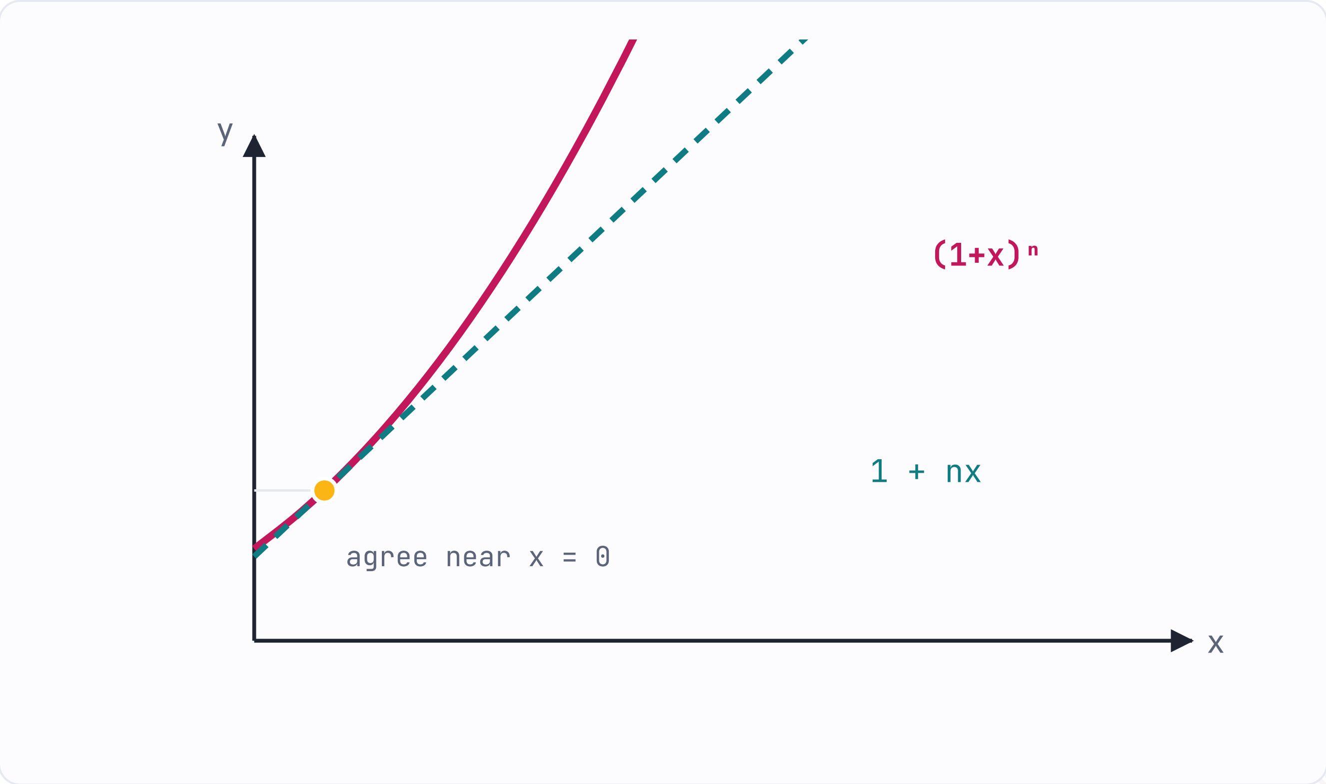
Two algebra tools dominate physics. First the quadratic formula, which kinematics ($s = ut + \frac{1}{2}at^2$) and projectile problems constantly produce; second, and far more important, the binomial approximation.

```
ax² + bx + c = 0 ⇒ x = [ -b ± √(b² - 4ac) ] / 2a
```

For $|x| \ll 1$, the binomial expansion collapses to a single linear line:

```
(1 + x)ⁿ ≈ 1 + nx (higher terms n(n-1)x²/2 … are negligible when x is tiny)
```

This one line powers gravitational potential at large distances, relativistic expansions, lensmaker corrections, error analysis, and any small-displacement problem. The whole art is spotting a small quantity and dropping its square.




*The binomial approximation is the tangent line to $(1+x)^n$ at $x = 0$. Near the origin the curve and its tangent are indistinguishable, which is why dropping $x^2$ is safe for small $x$.*

WORKED · SMALL CORRECTION

A pendulum's effective length changes by 2%. How does its period $T = 2\pi\sqrt{(L/g)}$ change?

```
T ∝ L^(1/2) ⇒ T' = T(1 + 0.02)^(1/2) ≈ T(1 + ½·0.02) = T(1.01)
```

The period rises by about 1%, half the length change. No calculator needed: that is the power of $(1+x)^n \approx 1+nx$.

▲ ADVANCED · KEEP THE SECOND TERM WHEN THE FIRST VANISHES

Sometimes the linear term cancels (a symmetric configuration) and you need the quadratic term: $(1+x)^n \approx 1 + nx + n(n-1)x^2/2$. This appears in the net field of a dipole, where leading terms cancel and the surviving term scales as $1/r^3$. Rule: expand one order beyond the lowest non-zero term, then stop.

⚠ TRAP

The approximation needs $|x| \ll 1$; students apply it when x is not small and get nonsense. The form must be $(1 + x)^n$: rewrite $(a + b)^n = a^n(1 + b/a)^n$ first, then approximate using b/a as the small quantity (valid only if $b \ll a$). Forgetting to factor out $a^n$ is the classic error.

◆ OUT OF THE BOX

The binomial approximation is the first-order Taylor series in disguise. Every linearise-the-physics move you meet in two years, from small oscillations to the SHM pendulum to the lens equation, is the same idea: replace a curve by its tangent near the operating point. Master this one habit and half of approximate-the-result questions become trivial.

M02

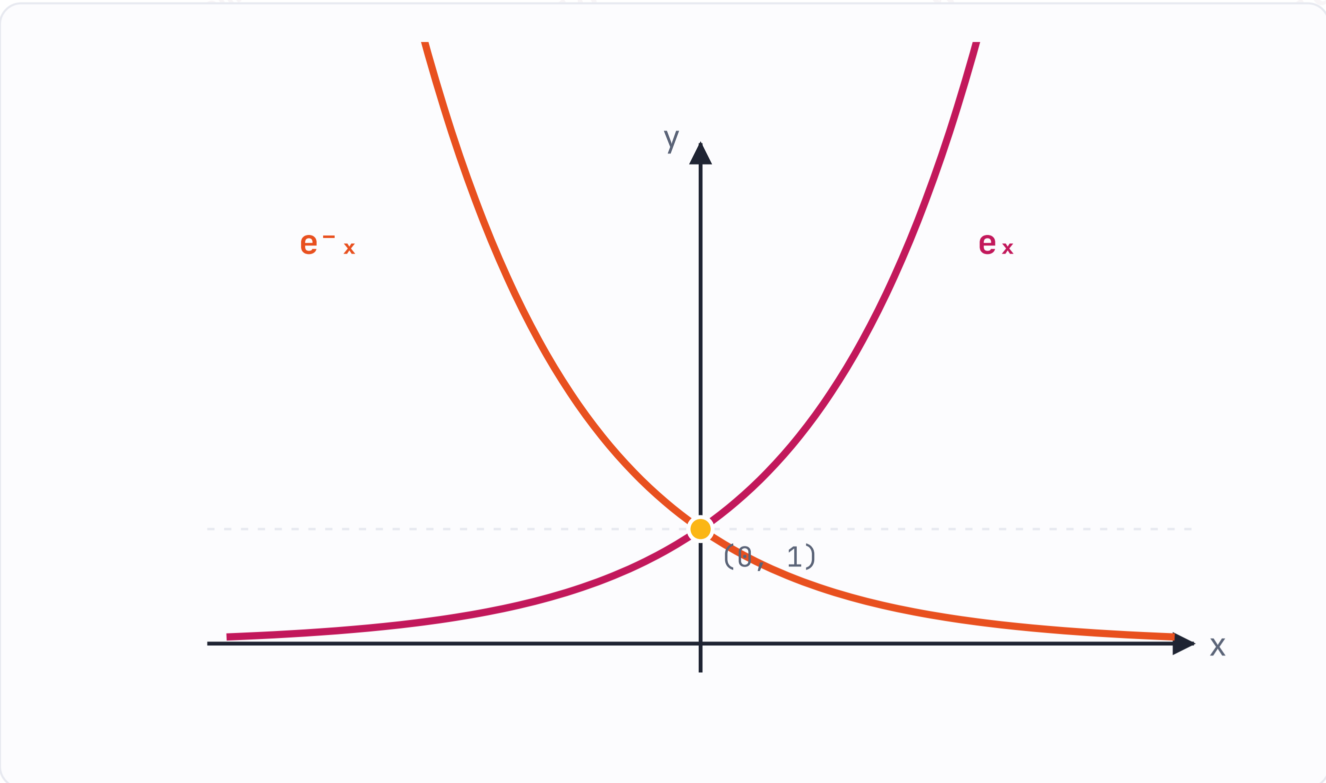
# Logarithms & Exponentials

Exponential growth and decay are everywhere: radioactive decay, capacitor charging, damped oscillations, atmospheric pressure with altitude. The key function is $e_x$ and its inverse, the natural logarithm ln x.

$\log(ab)=\log a + \log b$ · $\log(a/b)=\log a - \log b$ · $\log(a^n)=n \log a$

$\ln(e_x) = x$ · $e^{(\ln x)} = x$ · $\ln e = 1$ · change of base: $\log_a b = \ln b / \ln a$

The defining property of $e_x$: it is its own derivative. That is exactly why it describes any process whose rate of change is proportional to its current value, the signature of decay and growth laws.




*Exponential growth ($e_x$) and decay ($e^-{}_x$) are mirror images about the y-axis, both crossing y = 1 at x = 0. The decay curve approaches but never touches the axis, the basis of half-life, not full-life.*

**WORKED · RADIOACTIVE DECAY**

$N = N_0 e^{-\lambda t}$. Find the time for the sample to fall to half (the half-life).

$$N_0/2 = N_0 e^{-\lambda t} \Rightarrow ½ = e^{-\lambda t} \Rightarrow \ln(½) = -\lambda t$$
$$t_{½} = \ln 2 / \lambda \approx 0.693/\lambda$$

The 0.693 you memorise in Modern Physics is just ln 2, and now you know where it comes from.

**▲ ADVANCED · LOGARITHMIC DIFFERENTIATION PREVIEW**

When a quantity is a product or power of several variables (error propagation, $R = V/I$, or $T = 2\pi\sqrt{(L/g)}$), take ln of both sides first, then differentiate. ln converts products into sums, so fractional errors simply add: if $Q = A^x B^n$ then $\Delta Q/Q = |x|\cdot\Delta A/A + |y|\cdot\Delta B/B$. This is the engine behind every error-analysis question.

**⚠ TRAP**

In physics, log almost always means natural log (ln, base e) in calculus contexts, but base-10 in pH, decibels and Richter scales: check the context. And $\ln(a + b) \neq \ln a + \ln b$ (only products split). Misapplying the product rule to a sum is the single most common log error.

**◆ OUT OF THE BOX**

Why does e (≈ 2.718), an odd irrational number, run all of physics? Because it is the unique base where the curve's slope equals its height at every point. Nature does not choose e; e is simply what you get whenever a rate is proportional to an amount, which is the most common physical situation there is. The constant is not arbitrary, it is forced.

M03

# Trigonometry & the Small-Angle Approximation

Trig resolves vectors into components, describes waves and oscillations, and handles every inclined-plane problem. Lock the standard values and the quadrant signs.

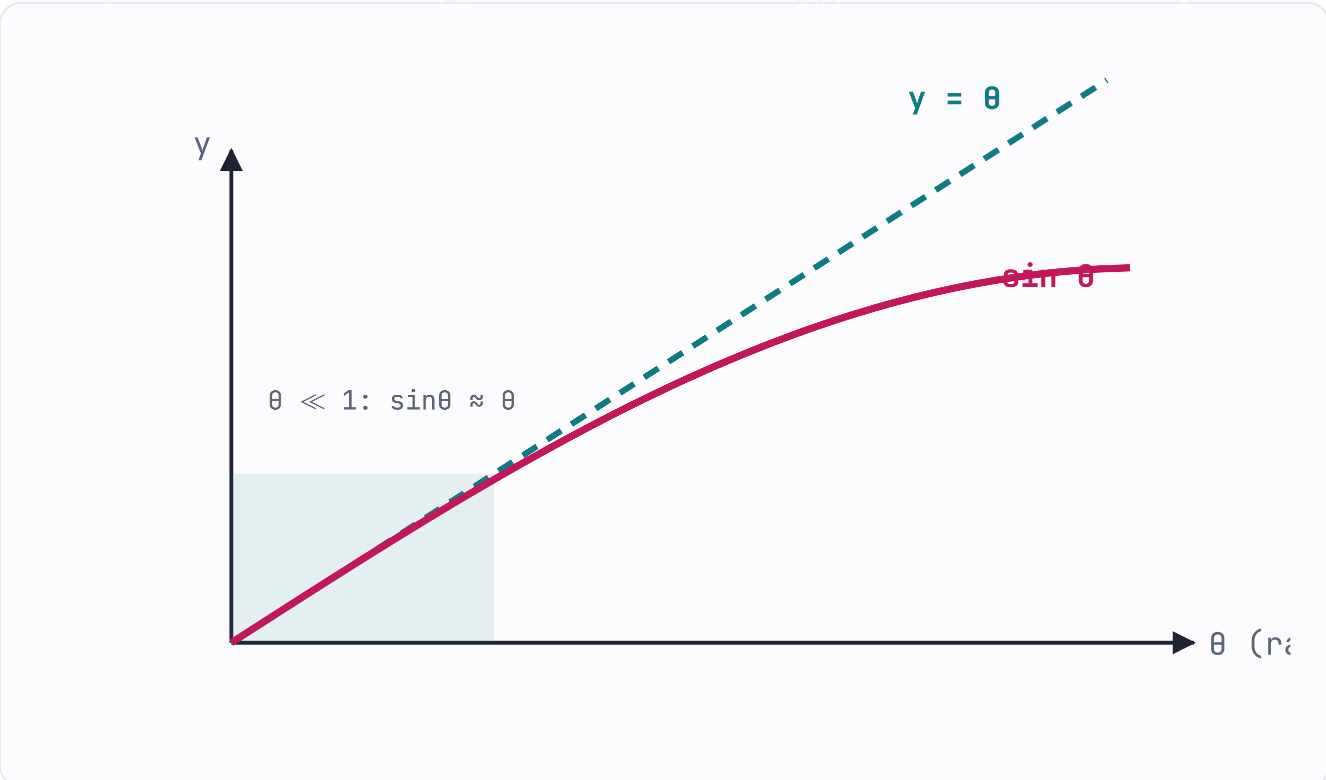
| θ | 0° | 30° | 37° | 45° | 53° | 60° | 90° |
|---|---|---|---|---|---|---|---|
| sin | 0 | 1/2 | 3/5 | 1/√2 | 4/5 | √3/2 | 1 |
| cos | 1 | √3/2 | 4/5 | 1/√2 | 3/5 | 1/2 | 0 |
| tan | 0 | 1/√3 | 3/4 | 1 | 4/3 | √3 | ∞ |

The 37°/53° pair (the 3-4-5 triangle) is a JEE and NEET favourite: examiners pick these angles precisely so the numbers come out clean. Memorise sin37° = 0.6, cos37° = 0.8.

$\sin^2\theta + \cos^2\theta = 1 \quad \cdot \quad 1 + \tan^2\theta = \sec^2\theta$

$\sin(A\pm B) = \sin A \cos B \pm \cos A \sin B \quad \cdot \quad \cos(A\pm B) = \cos A \cos B \mp \sin A \sin B$

small angle ($\theta$ in radians, $\theta \ll 1$): $\sin\theta \approx \theta \quad \cdot \quad \tan\theta \approx \theta \quad \cdot \quad \cos\theta \approx 1 - \theta^2/2$




*For small θ (in radians) the sine curve hugs the line y = θ. The approximation is what makes the simple pendulum simple: it linearises sinθ → θ to give SHM.*

**WORKED · WHY A PENDULUM IS SHM**

Restoring torque ∝ $\sin\theta$. For small swings, $\sin\theta \approx \theta$, so the restoring force becomes linear in $\theta$:

$$a = -(g/L)\sin\theta \approx -(g/L)\,\theta \Rightarrow \text{SHM with } \omega = \sqrt{g/L}$$

Drop the approximation and the motion is no longer simple harmonic: the period would depend on amplitude.

**▲ ADVANCED · RADIANS ARE NON-NEGOTIABLE**

Small-angle results hold only in radians ($\sin\theta \approx \theta$ is false in degrees). In calculus, $d/d\theta(\sin\theta) = \cos\theta$ is also a radian-only identity; the degree version carries an ugly $\pi/180$ factor. Always work physics calculus in radians. JEE buries this: a problem stated in degrees still needs radian conversion before any derivative or approximation.

**⚠ TRAP**

$\cos\theta \approx 1$ is the zeroth approximation; when you need the small change in cos (potential energy of a pendulum, $U = mgL(1-\cos\theta)$), you must use $\cos\theta \approx 1 - \theta^2/2$, giving $U \approx \frac{1}{2}mgL\theta^2$. Using $\cos\theta \approx 1$ there gives $U \approx 0$, which is wrong. Match the order of approximation to what the problem actually needs.

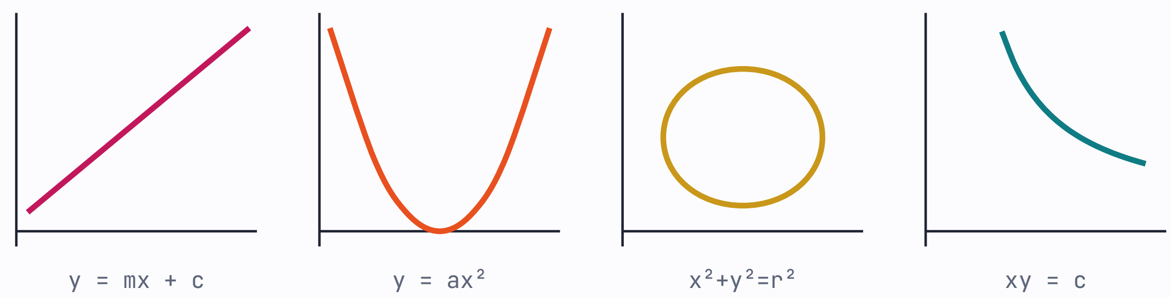
**◆ OUT OF THE BOX**

Notice the hierarchy: $\sin\theta \approx \theta$ (first order), $\cos\theta \approx 1 - \theta^2/2$ (second order). They are different orders because sine is odd and cosine is even: sine's leading correction is cubic, cosine's is quadratic. This is why a pendulum's displacement uses $\sin\theta \approx \theta$ but its energy needs the $\cos\theta$ term. Knowing which order to keep is a genuine JEE skill.

# Coordinate Geometry & Standard Curves

Physics graphs are coordinate geometry. Recognising a curve's shape from its equation lets you read physics off a graph instantly.

```
straight line: y = mx + c     parabola: y = ax² (or y² = 4ax)
circle: x² + y² = r²     ellipse: x²/a² + y²/b² = 1     rect. hyperbola: xy = const
```




*The four shapes you must recognise on sight. Boyle's law (PV = const) is a rectangular hyperbola; projectile height vs range is a parabola; a stress-strain elastic region is a straight line.*

PHYSICS ⇔ CURVE DICTIONARY

**Straight line** → v–t at constant a, Ohm's law V–I, Hooke's law F–x.

**Parabola** → x–t under constant acceleration, KE vs speed, projectile path.

**Rectangular hyperbola** → P–V isotherm (Boyle), intensity vs distance$^2$.

**Sinusoid** → SHM, AC voltage, waves.

▲ ADVANCED · LINEARISING TO EXTRACT CONSTANTS

When data fits $y = kx^n$, plotting ln y vs ln x gives a straight line of slope n and intercept ln k. For $y = A\,e^{-kx}$, plot ln y vs x: slope = −k. JEE experimental-physics questions reward turning a curve into a straight line so slope and intercept yield the physical constants. Always ask: what should I plot to get a straight line?

⚠ **TRAP**

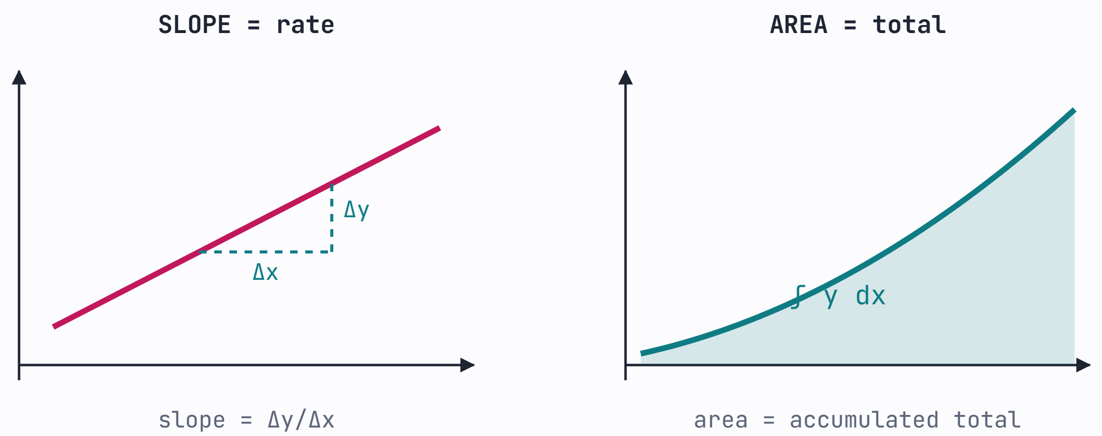
$y = ax^2$ and $y^2 = 4ax$ are both parabolas but oriented differently (opening up vs sideways). A v–t graph that is a straight line means constant acceleration, but an x–t straight line means constant velocity. The same shape means different physics on different axes: always read the axis labels first.

M05

# Functions & Graphs in Physics — Slope & Area

Two geometric ideas unlock kinematics and beyond: the slope of a graph and the area under it. These are the visual faces of differentiation and integration.

```
slope of a graph = rate of change = derivative (dy/dx)
area under a graph = accumulated total = integral (∫y dx)
```




*Left: the slope of a tangent gives an instantaneous rate (v–t slope = acceleration). Right: the area under the curve gives a total (v–t area = displacement).*

**THE KINEMATICS GRAPH CHAIN — KNOW THIS COLD**

**x–t graph:** slope = velocity.

**v–t graph:** slope = acceleration; area = displacement.

**a–t graph:** area = change in velocity.

Differentiate to go down the chain (x→v→a), integrate to go back up (a→v→x).

**▲ ADVANCED · SIGNED AREA & CONCAVITY**

Area below the axis is negative (velocity reversed → negative displacement contribution). For displacement use signed area; for distance travelled use the absolute area. Also: the concavity of an x–t graph reveals the sign of acceleration. Concave up = positive a, concave down = negative a, even where you cannot compute it. Reading concavity saves time on conceptual MCQs.

⚠ TRAP

A steep v–t graph means large acceleration, not large velocity. Students read high on the graph as fast-changing, but height is the value, slope is the rate. And the area under a v–t graph is displacement only if you respect sign; treating all area as positive confuses distance with displacement.

◆ OUT OF THE BOX

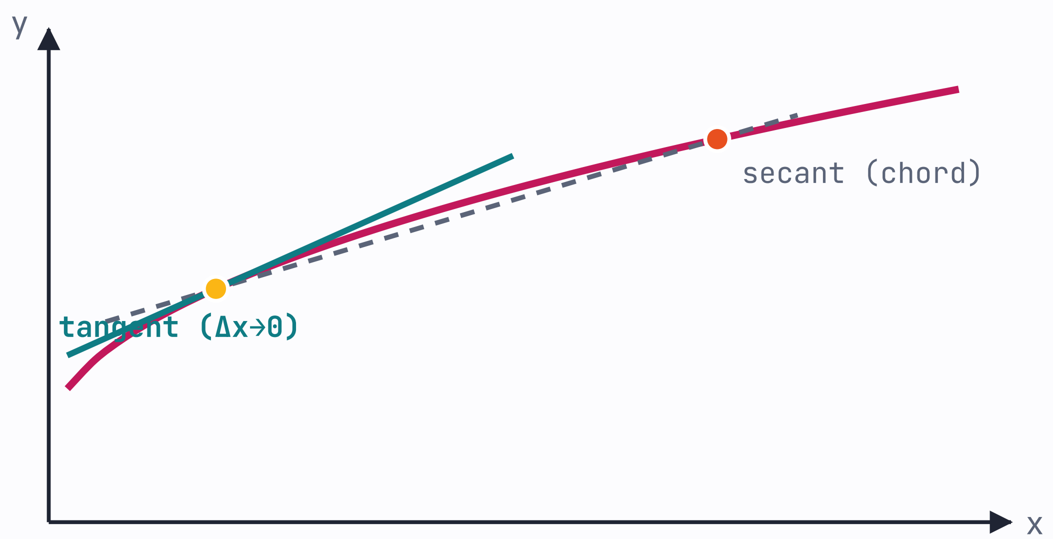
Slope and area are inverse operations, and that is the entire Fundamental Theorem of Calculus, stated visually. Take a v–t graph: the slope gives a, the area gives x; differentiation and integration undo each other. You are already doing calculus every time you read a motion graph, before you write a single dy/dx.

M06

# Differentiation — The Basics

The derivative dy/dx is the instantaneous rate of change of y with respect to x, the slope of the tangent at a point. It is defined as the limit of the slope of a chord as the two points merge.

$$dy/dx = \lim_{\Delta x \to 0} [\ f(x+\Delta x) - f(x)\ ] / \Delta x$$




*The derivative is the chord's slope in the limit where its two ends meet, turning an average rate into an instantaneous one.*

In practice you never use the limit. You use standard results:

| f(x) | df/dx | f(x) | df/dx |
|---|---|---|---|
| $x^n$ | $n \cdot x^{n-1}$ | sin x | cos x |
| constant | 0 | cos x | -sin x |
| $e_x$ | $e_x$ | tan x | $\sec^2 x$ |
| ln x | 1/x | $\sqrt{x}$ | $1/(2\sqrt{x})$ |

WORKED · VELOCITY & ACCELERATION FROM POSITION

A particle moves as $x(t) = 3t^3 - 2t^2 + 5t$. Find v and a.

```
v = dx/dt = 9t² - 4t + 5      a = dv/dt = 18t - 4
```

Differentiating once gives velocity; twice gives acceleration. The calculus does the work the constant-a equations of motion only approximate.

▲ ADVANCED · HIGHER DERIVATIVES & THEIR MEANING

The second derivative $d^2y/dx^2$ measures how the rate itself changes: acceleration is the second derivative of position. In SHM, $x = A \sin(\omega t)$ gives $a = -\omega^2 x$: acceleration proportional to displacement, opposite in sign, which is the definition of SHM. Recognising $a = -(\text{const})\cdot x$ as SHM straight from a differential equation is a core JEE move.

⚠ TRAP

dy/dx is one symbol, not a fraction you can split carelessly, though in physics we often treat dy and dx as separable (valid here, used constantly in integration). The derivative of a constant is 0, not the constant. And $d/dx(x^n) = nx^{n-1}$ even for negative and fractional n: $d/dx(1/x) = -1/x^2$, $d/dx(\sqrt{x}) = 1/(2\sqrt{x})$.

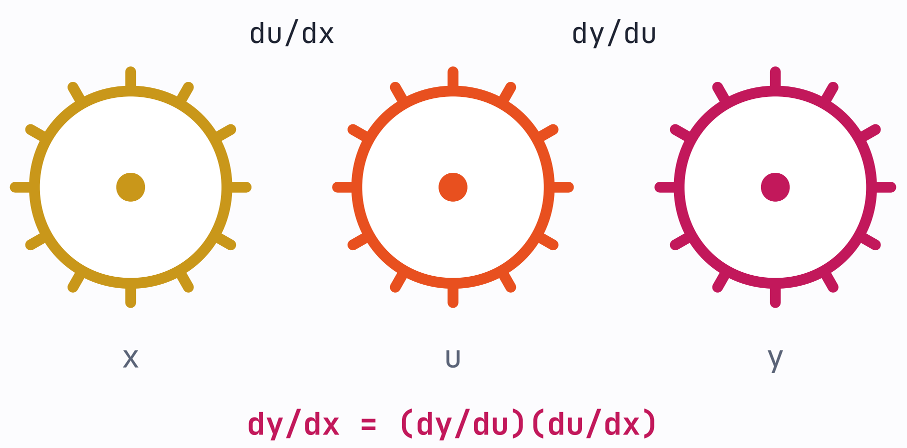
M07

# Rules of Differentiation

Four rules let you differentiate almost any physics expression.

```
sum: (u + v)′ = u′ + v′      product: (uv)′ = u′v + uv′
quotient: (u/v)′ = (u′v - uv′) / v²      chain: dy/dx = (dy/du)(du/dx)
```

The chain rule is the workhorse: any function of a function needs it, and physics is full of them, like $\sin(\omega t + \phi)$, $e^{-kt}$, and $(a^2 + x^2)^{3/2}$ in field problems.




*Think of the chain rule as meshed gears: the overall rate dy/dx is the product of the rate at each linkage. Identify the inner function u, differentiate the outer, then multiply by u′.*

**WORKED · CHAIN RULE IN SHM & DECAY**

$$(a)\ y = \sin(\omega t + \varphi).\ \text{Let } u = \omega t + \varphi.\ dy/dt = \cos(u)\cdot du/dt = \omega\cos(\omega t + \varphi)$$

$$(b)\ N = N_0 e^{-\lambda t}.\ dN/dt = N_0\cdot e^{-\lambda t}\cdot(-\lambda) = -\lambda N$$

In (b) the decay rate is proportional to the amount present, which is the decay law itself.

WORKED · PRODUCT RULE

Power delivered: $P = Fv$ where both F and v vary with time.

```
dP/dt = (dF/dt)v + F(dv/dt)
```

The rate of change of power has a force-changing piece and a speed-changing piece.

▲ ADVANCED · RELATED RATES & IMPLICIT DIFFERENTIATION

When two quantities are linked ($x^2 + y^2 = L^2$ for a sliding ladder, or $PV = nRT$), differentiate both sides with respect to time to relate their rates: $2x(dx/dt) + 2y(dy/dt) = 0$. Related-rates problems, a balloon inflating, a shadow lengthening, a rod sliding down a wall, all use this. The trick is differentiating a relationship, not an explicit function.

⚠ TRAP

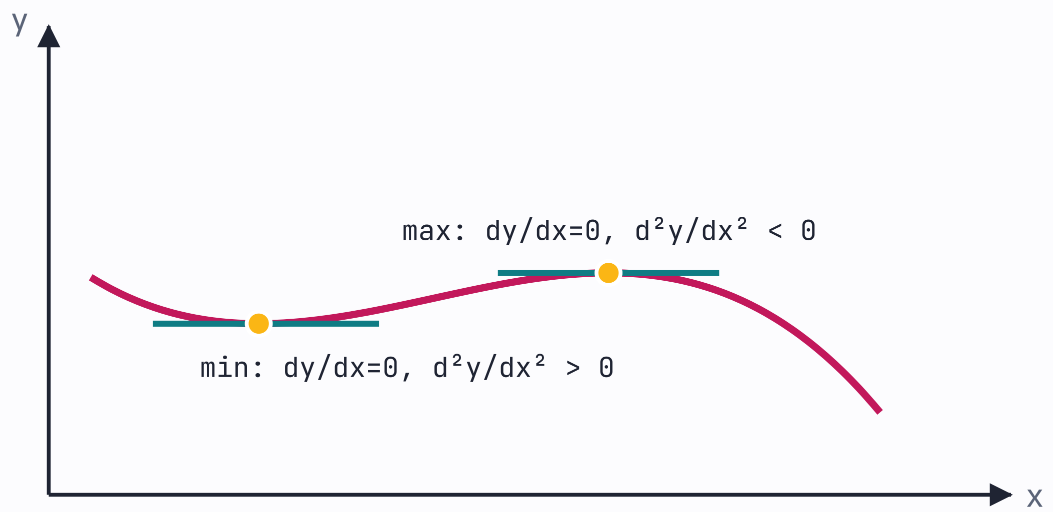
The product rule is not $(uv)' = u'v'$: that is the single most common calculus error. It is $u'v + uv'$. For the chain rule, students forget to multiply by the inner derivative du/dx (the $\times\omega$ in $\sin(\omega t)$). Missing the inner derivative silently halves or destroys your answer.

M08

# Maxima & Minima in Physics

At a maximum or minimum the tangent is horizontal, so the first derivative is zero. The sign of the second derivative tells you which: a negative value indicates a maximum (the curve frowns), and a positive value indicates a minimum (the curve smiles).

```
Step 1: dy/dx = 0 → solve for critical x
Step 2: d²y/dx² < 0 ⇒ maximum  ·  d²y/dx² > 0 ⇒ minimum
```




*At both turning points the tangent is flat (slope 0). The curvature, the sign of the second derivative, distinguishes a peak from a valley.*

**WORKED · MAXIMUM RANGE OF A PROJECTILE**

Range $R(\theta) = (v^2/g)\sin 2\theta$. Maximise over launch angle $\theta$:

```
dR/dθ = (v²/g)·2cos2θ = 0 ⇒ cos2θ = 0 ⇒ 2θ = 90° ⇒ θ = 45°
d²R/dθ² = -(4v²/g)sin2θ < 0 at θ=45° ⇒ confirmed maximum
```

Calculus proves the 45° result you otherwise just memorise, and the same method finds the optimal angle on an incline, where the answer is not 45°.

▲ ADVANCED · OPTIMISATION IS EVERYWHERE IN JEE

This method finds the angle for maximum range, the resistance for maximum power transfer ($R = r$), the speed for minimum time, the configuration for minimum potential energy (stable equilibrium), the radius giving maximum current. Whenever a problem says maximum, minimum, optimum or most economical, set the derivative to zero. Stable equilibrium is just a minimum of $U(x)$: $dU/dx = 0$ and $d^2U/dx^2 > 0$.

⚠ TRAP

$dy/dx = 0$ only locates a critical point: it could be a max, a min, or a point of inflexion. Always check the second derivative (or the sign change of the first). Also watch endpoints: on a closed interval the true maximum may sit at a boundary where the derivative is not zero, not at an interior turning point.

◆ OUT OF THE BOX

Nature is an optimiser. Light takes the path of least time (Fermat), systems settle at minimum potential energy, soap films minimise area, a hanging chain minimises energy. The entire framework of advanced physics, the principle of least action, is a maxima–minima problem. Setting a derivative to zero is not just a maths trick: it is how the universe seems to make its choices.

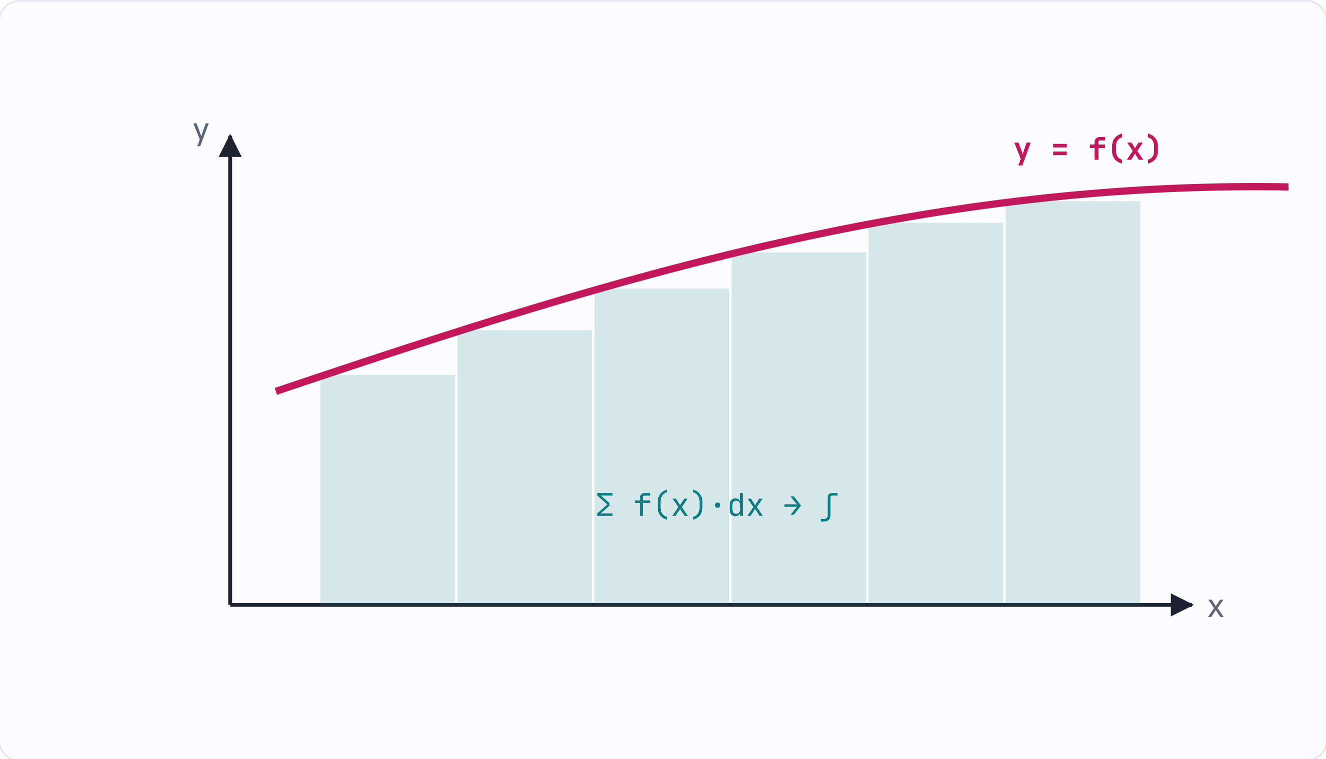
M09

# Integration — Antiderivatives & Area

Integration is differentiation run backwards: find the function whose derivative is the given one. It is also the area under a curve, and the two ideas are the same (the Fundamental Theorem of Calculus).

$$\int x^n\,dx = x^{n+1}/(n+1) + C \quad (n \neq -1)$$
$$\int (1/x)\,dx = \ln|x| + C \quad \cdot \quad \int e^x\,dx = e^x + C$$
$$\int \sin x\,dx = -\cos x + C \quad \cdot \quad \int \cos x\,dx = \sin x + C$$

The + C (constant of integration) matters in physics: it is fixed by an initial condition, the starting position, the initial velocity, the charge at t = 0.




*The integral sums infinitely many thin rectangles of height f(x) and width dx. As the strips shrink, the staircase becomes the exact area under the curve.*

WORKED · RECOVERING MOTION BY INTEGRATION

Acceleration $a = 6t$ (m/s$^2$), with $v = 4$ m/s and $x = 0$ at $t = 0$.

```
v = ∫a dt = 3t² + C₁ ; at t=0, v=4 ⇒ C₁ = 4 ⇒ v = 3t² + 4
x = ∫v dt = t³ + 4t + C₂ ; at t=0, x=0 ⇒ C₂ = 0 ⇒ x = t³ + 4t
```

Initial conditions pin down both constants. This is how you handle non-constant acceleration, where $v = u + at$ fails.

▲ ADVANCED · SUBSTITUTION & SEPARATION OF VARIABLES

Two techniques carry most JEE integrals. Substitution: for $\int f(g(x))g'(x)dx$, let $u = g(x)$, the reverse of the chain rule. Separation of variables: for $dv/dt = -kv$ (resistive motion) or $dv/dt = g - kv$ (terminal velocity), gather all $v$ on one side and $t$ on the other: $\int dv/(\dots) = \int dt$. This solves nearly every variable-force differential equation you will meet.

⚠ TRAP

Never forget the + C in an indefinite integral; in physics, dropping it loses the initial condition and gives wrong absolute values. The power rule $\int x^n dx = x^{n+1}/(n+1)$ fails for $n = -1$ (you would divide by zero); that case gives $\ln|x|$ instead. Both omissions are routinely punished.

M10

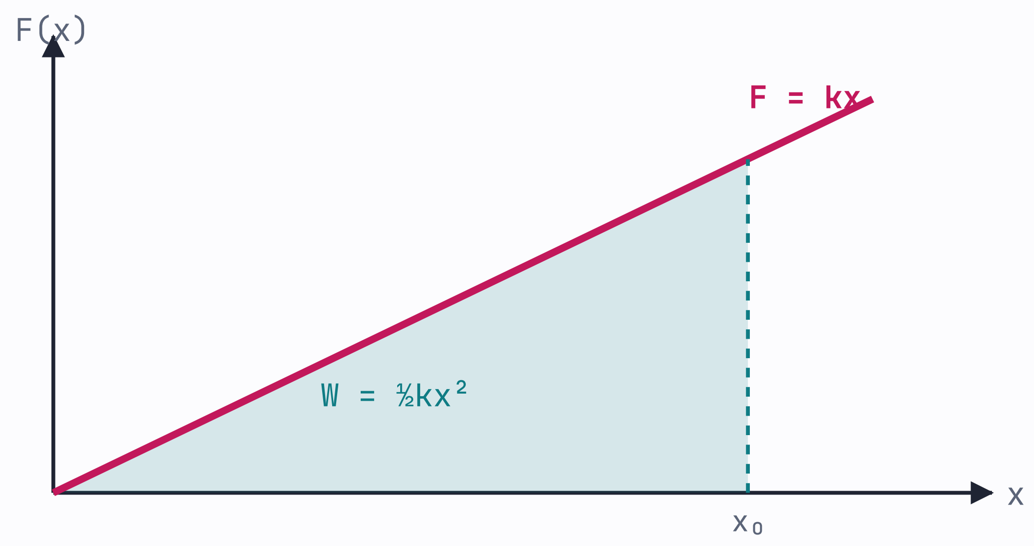
# Definite Integrals & Physical Applications

A definite integral evaluates the area between two limits, with no + C needed because it subtracts.

$$\int_a^b f(x)\,dx = F(b) - F(a) \quad \text{where } F \text{ is any antiderivative of } f$$

This is how physics totals a quantity that varies continuously: work done by a varying force, charge from a varying current, centre of mass, moment of inertia.

| PHYSICAL QUANTITY | AS A DEFINITE INTEGRAL |
|---|---|
| Displacement | $\int v\,dt$ |
| Work by variable force | $\int F\,dx$ |
| Charge through a wire | $\int i\,dt$ |
| Impulse | $\int F\,dt$ |
| Centre of mass | $(1/M) \int x\,dm$ |




*Work done stretching a spring is the area under the F = kx line, a triangle of area ½·x·(kx) = ½kx². The definite integral gives the familiar elastic PE directly.*

WORKED · WORK DONE BY A VARIABLE FORCE

A force $F = kx$ stretches a spring from 0 to $x_0$. Work done:

$$W = \int_0^{x_0} kx\,dx = k\left[x^2/2\right]_0^{x_0} = \tfrac{1}{2}kx_0^2$$

The stored elastic potential energy $\frac{1}{2}kx^2$ is a definite integral, not a memorised formula but a derived one.

▲ ADVANCED · SYMMETRY & THE DM TRICK

Two high-value moves. Symmetry: $\int_{-a}^{a}$ of an odd function = 0 (saves huge computation in field and flux problems). The dm method: for centre of mass, moment of inertia, or the field of a continuous body, write a tiny element $dm = \lambda dx$ (linear), $\sigma dA$ (areal) or $\rho dV$ (volume), then integrate over the body. Choosing the right element and limits is the entire skill in rigid-body and electrostatics integration.

⚠ TRAP

Get the limits in the right order and right variable: if you substitute $u = g(x)$, convert the limits to u-values (or back-substitute before plugging in). A definite integral can be negative or zero: area below the axis counts as negative, so the integral of a velocity that reverses gives net displacement, which may be less than the distance travelled.

◆ OUT OF THE BOX

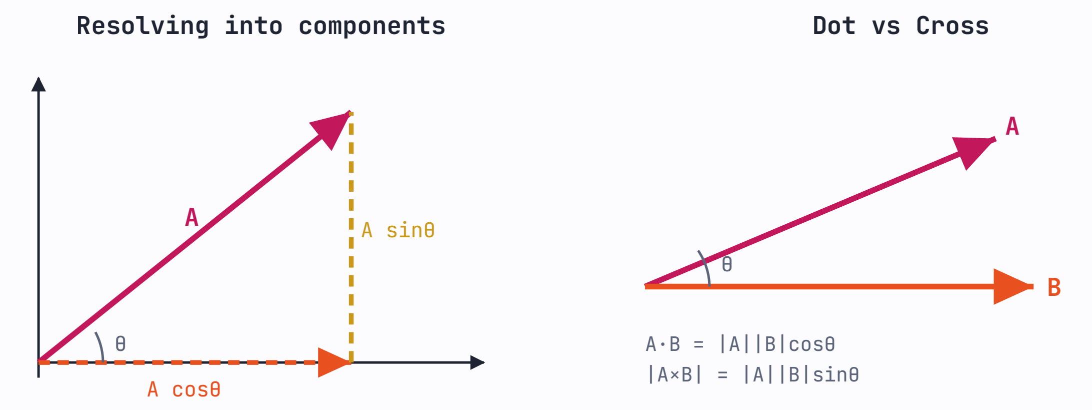
Notice how the same operation, integrate the rate to get the total, appears as work ($\int F\,dx$), impulse ($\int F\,dt$), charge ($\int i\,dt$) and displacement ($\int v\,dt$). They look like four formulae but they are one idea: accumulate a varying quantity over its variable. Once you see integration as continuous addition, you can build the right integral for a situation you have never seen before, which is exactly what JEE tests.

M11

# Vectors — The Essentials

A vector has magnitude and direction (displacement, velocity, force); a scalar has magnitude only (mass, time, energy). Vectors add tip-to-tail or by components.

```
components: Aₓ = A cosθ, A_y = A sinθ  ·  |A| = √(Aₓ² + A_y²)
dot (scalar): A·B = |A||B|cosθ = AₓBₓ + A_yB_y
cross (vector): |A×B| = |A||B|sinθ, direction by right-hand rule
```




*Left: any vector splits into perpendicular components via cosθ and sinθ. Right: the dot product peaks when vectors are parallel (cos), the cross product peaks when perpendicular (sin).*

WHERE EACH PRODUCT SHOWS UP

**Dot product (scalar):** Work $W = F \cdot d$, power $P = F \cdot v$, flux $\Phi = E \cdot A$. How much of one along the other.

**Cross product (vector):** Torque $\tau = r \times F$, angular momentum $L = r \times p$, magnetic force $F = qv \times B$. The perpendicular twist.

▲ ADVANCED · UNIT VECTORS & THE DETERMINANT CROSS PRODUCT

Write vectors in î, ĵ, k̂ form and compute the cross product as a 3×3 determinant:

$$A \times B = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

This gives both magnitude and direction in one step, essential for 3-D torque, magnetic force, and angular momentum problems where the right-hand rule alone is error-prone. Also know: A·(B×C) is the scalar triple product, equal to the volume of the parallelepiped, and it vanishes for coplanar vectors.

⚠ TRAP

The dot product is a scalar (a number); the cross product is a vector. Swapping them is a conceptual disaster: work is never a vector, torque is never a scalar. Also A×B = −B×A (order matters, anti-commutative), while A·B = B·A (order does not). Vectors add by the parallelogram or tip-to-tail rule: never add magnitudes arithmetically unless they are parallel.

◆ OUT OF THE BOX

The dot and cross products answer opposite questions. Dot asks how aligned (maximum when parallel, zero when perpendicular). Cross asks how perpendicular (zero when parallel, maximum when perpendicular). Together they decompose any interaction between two vectors into an along part and an across part, which is why one gives energy (along) and the other gives rotation (across). Two products, and between them they describe all of vector physics.

# ★ One-Page Formula Sheet

*Everything above, condensed. Print this page alone.*

### QUADRATIC & BINOMIAL

$x = [-b \pm \sqrt{b^2-4ac}] / 2a$

$(1+x)^n \approx 1 + nx \ (|x| \ll 1)$

next term: $+ \ n(n-1)x^2/2$

### LOGS & EXPONENTIALS

$\log(ab)=\log a+\log b$

$\log(a^n)=n \log a$

$\ln e = 1, \ e^{\ln x}=x$

half-life $t_{1/2} = \ln 2/\lambda = 0.693/\lambda$

### TRIG VALUES

$\sin 37°=0.6, \ \cos 37°=0.8$

$\sin 53°=0.8, \ \cos 53°=0.6$

$\sin 45°=\cos 45°=1/\sqrt{2}$

$\sin^2\theta+\cos^2\theta=1, \ 1+\tan^2\theta=\sec^2\theta$

### SMALL ANGLE (RADIANS)

$\sin\theta \approx \theta, \ \tan\theta \approx \theta$

$\cos\theta \approx 1 - \theta^2/2$

### STANDARD CURVES

line: $y=mx+c$

parabola: $y=ax^2$

circle: $x^2+y^2=r^2$

rect. hyperbola: $xy=\text{const}$

### DERIVATIVES

$d(x^n)=n \ x^{n-1}$

$d(\sin x)=\cos x, \ d(\cos x)=-\sin x$

$d(\tan x)=\sec^2 x$

$d(e_x)=e_x, \ d(\ln x)=1/x$

### RULES OF DIFFERENTIATION

$(uv)' = u'v + uv'$

$(u/v)' = (u'v-uv')/v^2$

chain: $dy/dx=(dy/du)(du/dx)$

### MAXIMA / MINIMA

$dy/dx = 0 \rightarrow$ critical pt

$d^2y/dx^2 < 0 \rightarrow$ max

$d^2y/dx^2 > 0 \rightarrow$ min

### INTEGRALS

$\int x^n dx = x^{n+1}/(n+1)+C \ (n\neq-1)$

$\int(1/x)dx = \ln|x|+C$

$\int e_x dx = e_x+C$

$\int \sin x \, dx=-\cos x, \ \int \cos x \, dx=\sin x$

$\int_a^b f \, dx = F(b)-F(a)$

### CALCULUS IN KINEMATICS

$v = dx/dt, \ a = dv/dt$

$x = \int v \, dt, \ v = \int a \, dt$

v-t slope=a, v-t area=displacement

### VECTORS

$A_x=A \cos\theta, \ A_y=A \sin\theta$

$|A|=\sqrt{A_x^2+A_y^2}$

$A\cdot B=|A||B|\cos\theta$ (scalar)

$|A\times B|=|A||B|\sin\theta$ (vector, $\perp$)

$A\times B = -B\times A$

### COMMON PHYSICS INTEGRALS

displacement $= \int v \, dt$

work $= \int F \, dx$

impulse $= \int F \, dt$

charge $= \int i \, dt$

spring PE $= \int kx \, dx = \frac{1}{2}kx^2$

# Logic Bloom

*Understand through games. Score through practice.*

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