LOGIC BLOOM · JEE MATHEMATICS

Basic Maths *for JEE*

Foundation Notes · **Grade 10 → JEE bridge**

Logarithms, inequalities, identities, modulus, the wavy curve method & more: every fundamental you need before the real JEE syllabus begins.

8 Topics | Every Concept with Examples | Graphs | Formula Sheet

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Understand through games.
Score through practice.

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Sets N–R, interval notation

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Everything for revision day

1 Number System & Intervals

Before any algebra, you must know which numbers you are working with and how to write a set of them cleanly. JEE solutions are almost always stated as intervals, so fluency here saves marks everywhere.

Set	Symbol	Examples
Natural numbers	N	1, 2, 3, ...
Whole numbers	W	0, 1, 2, 3, ...
Integers	Z	..., −2, −1, 0, 1, 2, ...
Rational numbers	Q	p/q with $q \neq 0$ (e.g. 3/4, −5, 0.25)
Irrational numbers	Q′	$\sqrt{2}$, π, e : non-terminating, non-repeating
Real numbers	R	everything on the number line ($Q \cup Q'$)

INTERVAL NOTATION: LEARN BOTH FORMS

An interval is a connected piece of the number line. A **square bracket** [or] means the endpoint is *included*; a **round bracket** (or) means it is *excluded*. Infinity always takes a round bracket.

- $(a, b) = \{x : a < x < b\}$, open, neither end.
- $[a, b] = \{x : a \le x \le b\}$, closed, both ends.
- $[a, b)$ and $(a, b]$, half-open.
- $(a, \infty) = \{x : x > a\}$ · $(-\infty, b] = \{x : x \le b\}$.

EXAMPLE · WRITING A SOLUTION SET

“All real x with $x \ge -2$ and $x < 5$” is written **$[-2, 5)$**. The −2 is included (square), the 5 is excluded (round). In set form: $\{x \in R : -2 \le x < 5\}$.

2 Indices (Exponents) & Surds

Powers and roots are the grammar of every later formula. The laws below are used without comment in physics, logarithms and calculus, so they must be automatic.

$$a^m \cdot a^n = a^{m+n} \quad \cdot \quad a^m/a^n = a^{m-n} \quad \cdot \quad (a^m)^n = a^{mn}$$
$$(ab)^n = a^n b^n \quad \cdot \quad a^0 = 1\ (a \neq 0) \quad \cdot \quad a^{-n} = 1/a^n$$
$$a^{1/n} = \sqrt[n]{a} \quad \cdot \quad a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

SURDS: RATIONALISING THE DENOMINATOR

A surd is an irrational root like $\sqrt{2}$ or $\sqrt{5}$. To remove a surd from a denominator, multiply by the **conjugate**. For $1/(\sqrt{a} + \sqrt{b})$, multiply top and bottom by $(\sqrt{a} - \sqrt{b})$:

$$1/(\sqrt{a} + \sqrt{b}) \times (\sqrt{a} - \sqrt{b})/(\sqrt{a} - \sqrt{b}) = (\sqrt{a} - \sqrt{b})/(a - b)$$

EXAMPLE · RATIONALISE

Simplify $1/(\sqrt{5} - \sqrt{3})$. Multiply by the conjugate $(\sqrt{5} + \sqrt{3})$:

$$(\sqrt{5} + \sqrt{3}) / [(\sqrt{5})^2 - (\sqrt{3})^2] = (\sqrt{5} + \sqrt{3})/(5 - 3) = (\sqrt{5} + \sqrt{3})/2$$

EXAMPLE · FRACTIONAL POWERS

Evaluate $32^{3/5}$. Write $32 = 2^5$, so $32^{3/5} = (2^5)^{3/5} = 2^3 =$ **8**. Always express the base as a prime power first; the exponent rule then does the work.

3 Logarithms

A logarithm is just an exponent in disguise. $\log_a N$ answers the question: “to what power must I raise a to get N?” This single idea unlocks exponential equations, and it is everywhere in JEE.

Definition: $\log_a N = x \Leftrightarrow a^x = N \quad (a > 0,\ a \neq 1,\ N > 0)$

$\log_a 1 = 0 \quad \cdot \quad \log_a a = 1 \quad \cdot \quad a^{\log_a N} = N$

THE LAWS OF LOGARITHMS

- **Product:** $\log_a(MN) = \log_a M + \log_a N$
- **Quotient:** $\log_a(M/N) = \log_a M - \log_a N$
- **Power:** $\log_a(M^p) = p \cdot \log_a M$
- **Change of base:** $\log_a N = \log_b N / \log_b a = (\ln N)/(\ln a)$

Two common bases: **log** usually means base 10, **ln** means base e (≈ 2.718).

EXAMPLE · EVALUATE

Find $\log_2 8$ and $\log_3 81$. Since $8 = 2^3$, $\log_2 8 =$ **3**. Since $81 = 3^4$, $\log_3 81 =$ **4**. Read every log as “what power gives this number?”

EXAMPLE · SOLVE A LOG EQUATION

Solve $\log_2 x + \log_2(x-2) = 3$.

Combine using the product law, then convert to exponential form:

$$\log_2[x(x-2)] = 3 \Rightarrow x(x-2) = 2^3 = 8 \Rightarrow x^2 - 2x - 8 = 0 \Rightarrow (x-4)(x+2) = 0$$

So $x = 4$ or $x = -2$. **Reject $x = -2$** (the log of a negative number is undefined; we need $x > 2$). Answer: **$x = 4$**. Always check the domain after solving a log equation.

⚠ DOMAIN IS NON-NEGOTIABLE

Inside any logarithm the argument must be strictly positive, and the base must be positive and not 1. Many JEE questions are won purely by discarding roots that violate this. After solving, substitute back and keep only the valid values.

4

Algebraic Identities

Identities let you expand or factor instantly instead of multiplying term by term. Memorise the core set; they recur in coordinate geometry, binomial theorem and trigonometry.

$(a + b)^2 = a^2 + 2ab + b^2$ · $(a - b)^2 = a^2 - 2ab + b^2$

$a^2 - b^2 = (a + b)(a - b)$

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ · $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ · $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

THE TWO IDENTITIES JEE LOVES

From $(a+b+c)^2$ you get $a^2+b^2+c^2 = (a+b+c)^2 - 2(ab+bc+ca)$, used to find one symmetric sum from another. And if $a + b + c = 0$, the cube identity collapses to the beautiful result **$a^3 + b^3 + c^3 = 3abc$**, a frequent shortcut.

EXAMPLE · USE A SYMMETRIC IDENTITY

If $a + b = 5$ and $ab = 6$, find $a^2 + b^2$ and $a^3 + b^3$.

$a^2 + b^2 = (a+b)^2 - 2ab = 25 - 12 = 13$

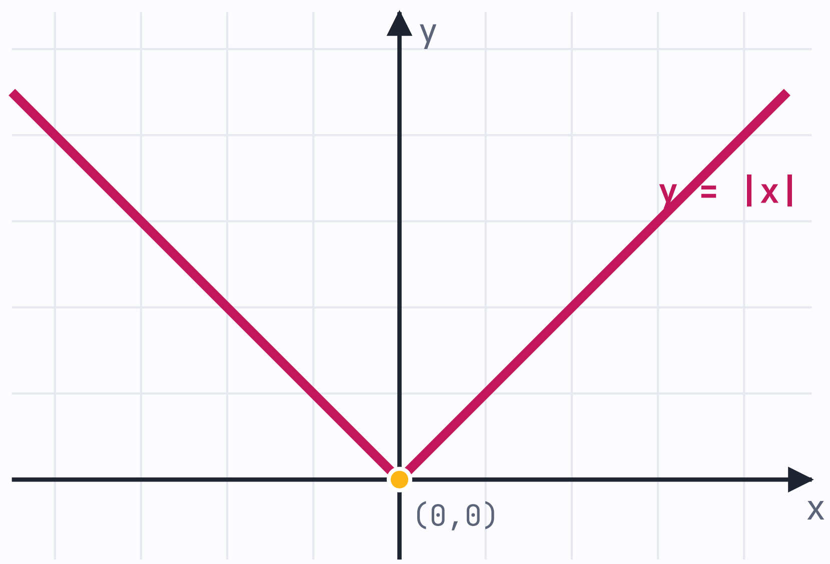
$a^3 + b^3 = (a+b)^3 - 3ab(a+b) = 125 - 3·6·5 = 125 - 90 = 35$

You never needed the actual values of a and b; symmetric identities give the answer directly.

5 Modulus (Absolute Value)

The modulus $|x|$ is the distance of x from 0 on the number line, so it is never negative. It is defined piecewise, and that piecewise nature is the key to every modulus problem.

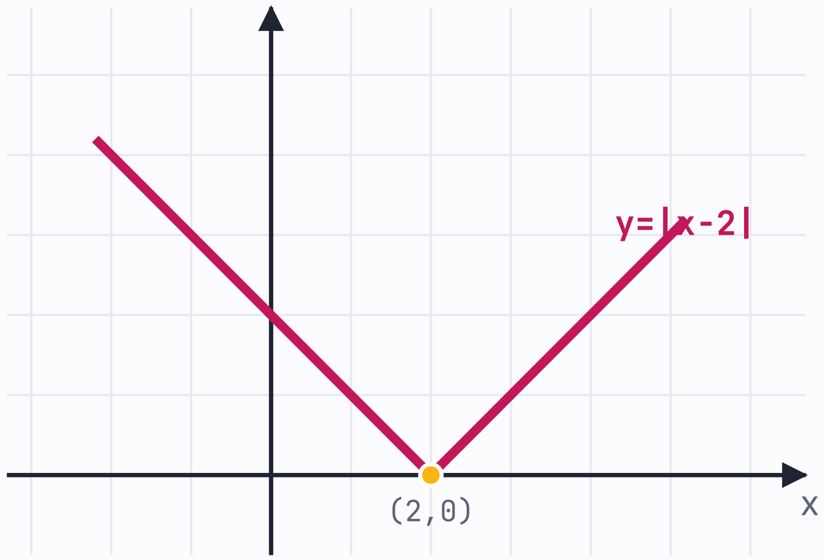
$$|x| = x \text{ if } x \geq 0 \quad \cdot \quad |x| = -x \text{ if } x < 0$$
$$|x| \geq 0 \quad \cdot \quad |x|^2 = x^2 \quad \cdot \quad \sqrt{(x^2)} = |x| \quad \cdot \quad |xy| = |x||y|$$



$y = |x|$ is a V with its corner at the origin: the line $y = x$ for $x \geq 0$ and $y = -x$ for $x < 0$, reflected into the upper half-plane.

SOLVING MODULUS EQUATIONS & INEQUALITIES (A > 0)

- $|x| = a \Rightarrow x = a$ or $x = -a$.
- $|x| < a \Rightarrow -a < x < a$ (a single interval: distance from 0 is small).
- $|x| > a \Rightarrow x < -a$ or $x > a$ (two pieces: distance from 0 is large).
- $|x - c| < a \Rightarrow c - a < x < c + a$ (within distance a of c).



$y = |x - 2|$ is the basic V slid 2 units right: the corner moves to (2, 0). In general $y = |x - c|$ has its corner at (c, 0).

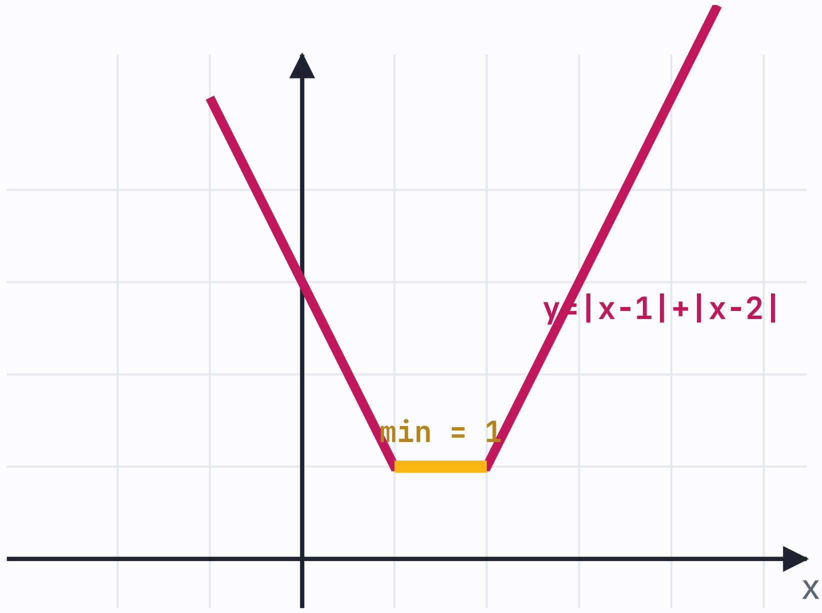
EXAMPLE · MODULUS EQUATION

Solve $|x - 3| = 5$. The distance from x to 3 is 5, so $x - 3 = 5$ or $x - 3 = -5$, giving **$x = 8$ or $x = -2$**.

EXAMPLE · MODULUS INEQUALITY

Solve $|x - 2| < 3$. This says x is within distance 3 of 2:

```
-3 < x - 2 < 3 ⇒ -1 < x < 5 ⇒ x ∈ (-1, 5)
```



$y = |x - 1| + |x - 2|$ has a flat valley of height 1 across [1, 2] (its minimum) and rises with slope 2 outside. Sums of moduli give such flat-bottomed graphs.

▲ SUM OF MODULI: THE FLAT MINIMUM

An expression like $|x - 1| + |x - 2|$ measures total distance from x to the points 1 and 2. Its minimum value equals the gap between the points (here $2 - 1 = 1$) and is attained for *all* x *between* them. Recognising this beats brute-force casework.

6 Inequalities

An inequality asks for a *range* of x, not a single value. The rules mirror equations with one crucial exception, plus two classical results JEE leans on.

⚠ THE RULES, AND THE ONE THAT FLIPS

You may add or subtract any quantity from both sides freely, and multiply or divide by a **positive** number freely. But multiplying or dividing by a **negative** number **reverses** the inequality sign. This single rule is the most common source of lost marks.

Example: from $-2x < 6$, dividing by -2 flips it to $x > -3$ (not $x < -3$).

EXAMPLE · LINEAR INEQUALITY

Solve $3 - 2x \geq 7$. Subtract 3: $-2x \geq 4$. Divide by -2 and **flip**: $x \leq -2$, i.e. $x \in (-\infty, -2]$.

EXAMPLE · QUADRATIC INEQUALITY

Solve $x^2 - 5x + 6 < 0$. Factor: $(x - 2)(x - 3) < 0$. A product is negative between its roots, so $\mathbf{2 < x < 3}$, i.e. $x \in (2, 3)$. (For "> 0" the answer would be *outside* the roots.)

★ AM – GM INEQUALITY (CLASSICAL, HIGH-YIELD)

For non-negative reals, the arithmetic mean is at least the geometric mean:

$$(a + b)/2 \geq \sqrt{ab} \quad \text{(equality iff } a = b)$$

$$\text{more generally: } (a_1+\dots+a_n)/n \geq (a_1\dots a_n)^{1/n}$$

This is the standard tool for "find the minimum value" problems where calculus is not wanted.

EXAMPLE · AM-GM MINIMUM

For $x > 0$, find the minimum of $x + 1/x$. By AM–GM on the two positive terms x and $1/x$:

$$(x + 1/x)/2 \geq \sqrt{x \cdot 1/x} = 1 \Rightarrow x + 1/x \geq 2$$

Minimum value is **2**, attained when $x = 1/x$, i.e. $x = 1$.

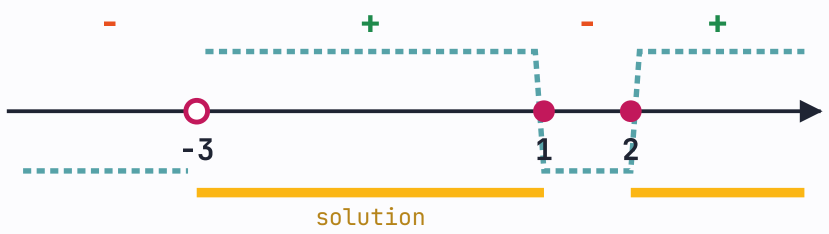
7

The Wavy Curve Method

This is the master technique for any inequality of the form (product of factors)/(product of factors) compared with 0. It replaces all casework with one clean sign diagram, and it is indispensable for JEE.

THE METHOD, STEP BY STEP

1. Make every factor of the form $(x - r)$ with a *positive* coefficient of x, and bring everything to one side so the inequality reads (...) > 0, < 0, ≥ 0 or ≤ 0.
2. Mark all critical points (roots of numerator and denominator) on the number line. Denominator roots are *always open* (the expression is undefined there).
3. Start from the far right, where the expression is **positive**, and draw a wave. At each simple factor the sign **changes**; at a factor of *even* power the sign **stays the same** (the curve touches and turns back).
4. Read off the intervals matching the required sign.



Wavy curve for $(x - 1)(x - 2)/(x + 3) \geq 0$. Critical points -3 (open, from the denominator), 1 and 2 (closed). Signs alternate; the gold bars mark the solution.

EXAMPLE · WAVY CURVE

Solve $(x - 1)(x - 2)/(x + 3) \geq 0$.

Critical points: $x = 1, 2$ (numerator, *included* since $\geq$) and $x = -3$ (denominator, *excluded*). Place them and assign signs from the right (+), alternating: for $x > 2$ positive; $(1, 2)$ negative; $(-3, 1)$ positive; $x < -3$ negative.

```
solution: (-3, 1] ∪ [2, ∞)
```

Note -3 is excluded (round) but 1 and 2 are included (square). Mixing these up is the classic slip.

▲ REPEATED FACTORS: THE SIGN DOES NOT FLIP

For $(x-1)^2(x-3) < 0$, the factor $(x-1)^2$ has even power, so the curve *touches* the axis at $x = 1$ without changing sign. The sign only flips at $x = 3$. Working it through gives the solution $x < 3$ with $x \neq 1$, i.e. $(-\infty, 1) \cup (1, 3)$.

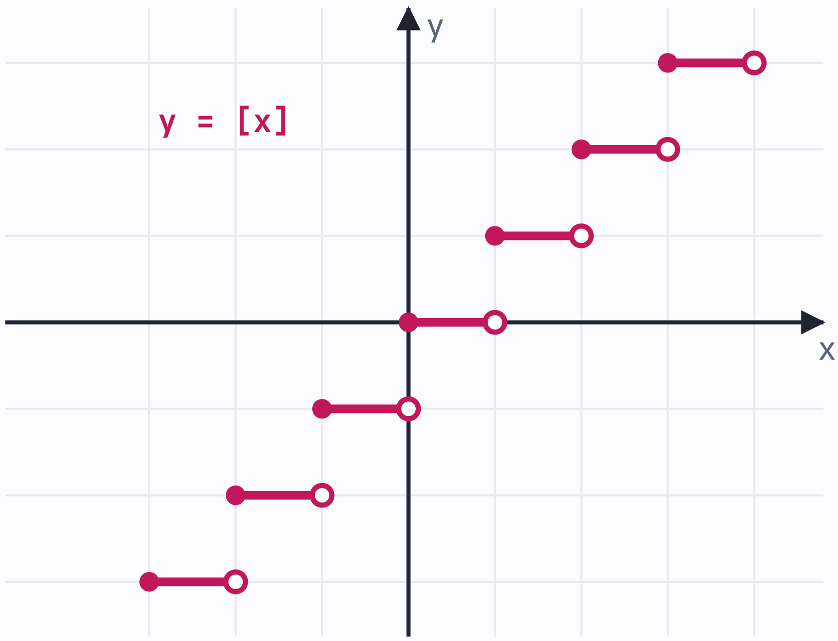
8 Greatest Integer & Fractional Part Functions

Two “special” functions that appear constantly in JEE limits, graphs and equations. Both are defined through the integers, and both are best understood from their graphs.

```
Greatest integer (floor): [x] = greatest integer ≤ x
e.g. [2.7] = 2   ·   [5] = 5   ·   [-2.3] = -3  (careful with negatives!)
Fractional part: {x} = x - [x],  always in [0, 1)
```

⚠ THE NEGATIVE-NUMBER TRAP

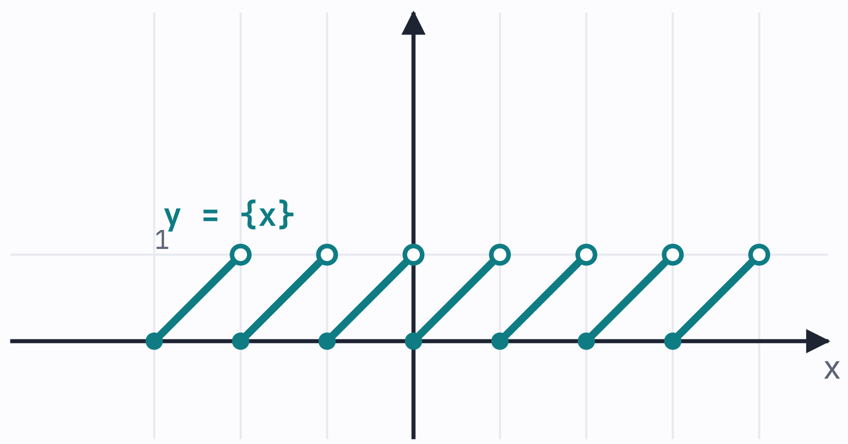
[x] rounds **down** (towards $-\infty$), not towards zero. So $[-2.3] = -3$, because -3 is the greatest integer that is still ≤ -2.3. Students routinely write -2, which is the single most common error with this function.



y = [x] is a staircase. On each interval [n, n+1) the value is the integer n: filled dot on the left (included), hollow dot on the right (excluded). The jump at every integer is 1.

KEY PROPERTIES

- $[x] = n \Leftrightarrow n \le x < n + 1$ (n an integer).
- $[x + n] = [x] + n$ for any integer n.
- $x = [x] + \{x\}$ always: integer part plus fractional part.
- $0 \le \{x\} < 1$, and $\{x\} = 0$ exactly when x is an integer.



$y = \{x\}$ is a sawtooth: it climbs from 0 up to (but not including) 1 on each unit interval, then drops back to 0 at the next integer. It is periodic with period 1.

EXAMPLE · EVALUATE

Find [3.9], [−1.2], {3.9} and {−1.2}. We get [3.9] = 3 and [−1.2] = −2. Then {3.9} = 3.9 − 3 = **0.9**, and {−1.2} = −1.2 − (−2) = **0.8**. The fractional part is always positive, even for negative x.

★ Basic Maths: One-Page Formula Sheet

Everything for revision day. Print this alone.

NUMBER SETS

$N \subseteq W \subseteq Z \subseteq Q \subseteq R$

Q' = irrationals ($\sqrt{2}$, π, e)

[ ] included · ( ) excluded

∞ always ( )

INDICES

$a^m a^n = a^{m+n}$ · $a^m/a^n = a^{m-n}$

$(a^m)^n = a^{mn}$ · $(ab)^n = a^n b^n$

$a^0 = 1$ · $a^{-n} = 1/a^n$

$a^{m/n} = \sqrt[n]{a^m}$

LOGARITHMS

$\log_a N = x \Leftrightarrow a^x = N$

$\log(MN) = \log M + \log N$

$\log(M/N) = \log M - \log N$

$\log(M^p) = p \log M$

$\log_a N = \ln N / \ln a$

IDENTITIES

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

$a^2 - b^2 = (a+b)(a-b)$

$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

$(a+b+c)^2 = \Sigma a^2 + 2\Sigma ab$

$a+b+c=0 \Rightarrow a^3+b^3+c^3 = 3abc$

MODULUS

$|x| = x$ $(x \geq 0)$, $-x$ $(x<0)$

$|x| = a \Rightarrow x = \pm a$

$|x| < a \Rightarrow -a < x < a$

$|x| > a \Rightarrow x < -a$ or $x > a$

$\sqrt{x^2} = |x|$

INEQUALITIES

$\times/\div$ by negative $\Rightarrow$ flip sign

quad <0: between roots

quad >0: outside roots

AM $\geq$ GM: $(a+b)/2 \geq \sqrt{ab}$

$x + 1/x \geq 2$ $(x>0)$

WAVY CURVE

factors $(x-r)$, +ve x-coeff

denominator roots: open

start + from far right

simple root: sign flips

even power: sign stays

GREATEST INTEGER

$[x]$=greatest integer $\leq x$

$[-2.3] = -3$ (rounds down)

$[x] = n \Leftrightarrow n \leq x < n+1$

$[x+n] = [x] + n$

FRACTIONAL PART

$\{x\} = x - [x] \in [0,1)$

$x = [x] + \{x\}$

$\{x\} = 0 \Leftrightarrow x$ integer

period 1 sawtooth

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